

# Teaching Through Problems Worth Solving

-Grade 8 (Version 3.0)-

Inquiry-based, Curriculum-linked, Differentiated Math Problems for Grade 8



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With thanks to the ATA Educational Trust for their financial support.

*A 21st Century Learning Promise: I promise to do all I can to keep the spark of curiosity, creativity, and learning alive in every child; to help all children discover their talents, develop their passions, deepen their understanding, and apply all this to helping others, and to creating a better world for us all.*

*-author unknown*

[Link to Thinking Classroom Problems](#)

[Link to Curricular Problems](#)

[Link to New Problems](#)

***The inspiration for this compilation of problems has come from many sources.***

***Thank you to:***

***Peter Liljedahl, David Pimm, Nathalie Sinclair, and Rina Zazkis  
Grande Prairie Master of Education Cohort, Simon Fraser University, 2012-2014  
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***Please go to [www.aliciaburdess.com](http://www.aliciaburdess.com) for a free digital copy and updated version!***

## **Read This First**

This resource is the result of a year-long collaborative project to identify and compile problems that align with the grade 8 curriculum outlined by the Alberta Mathematics Program of Studies (2007). It is an initial attempt to answer our essential question, **“How can teaching through problem solving engage every student and drive learning forward?”**

This resource is not meant as a bank of worksheets to be given arbitrarily to students. Rather, it is designed to be a journey through problem solving for the entire math classroom. Problems worth solving take time. Some problems may take only one block, others will take longer. Use your professional judgment to choose your problems, guide your teaching, and facilitate student learning. The focus is meant to be on the experience of the problem solving process - the thinking, the connections, and the understanding. Sample solutions are provided as a single example of many possible problem solving strategies. Our intent is for you the teacher to be deeply involved in the problem solving process with your students and hopefully with your colleagues. Take risks, make mistakes, and don't worry as much about the destination as about the journey. Complement these problems with mini-lessons, games, and projects to teach the Grade 8 Program of Studies.

This is our second draft. Some outcomes have more problems linked to them than others. This project is ongoing; it will continue to be tested with students and improved. We also have plans to translate the problems into French in the near future.

## A Problem Solving Classroom

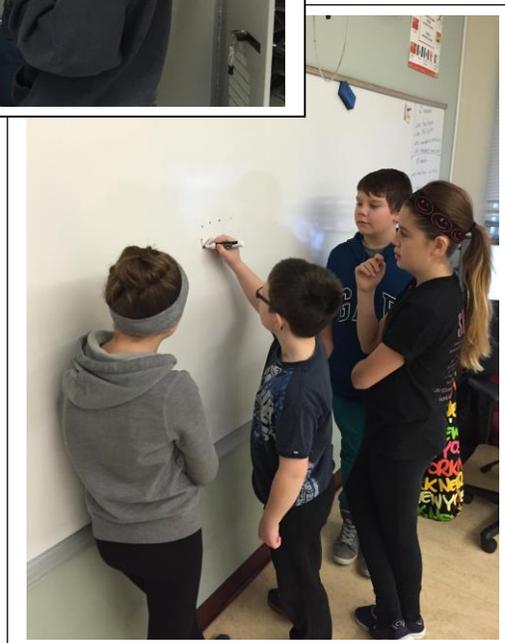
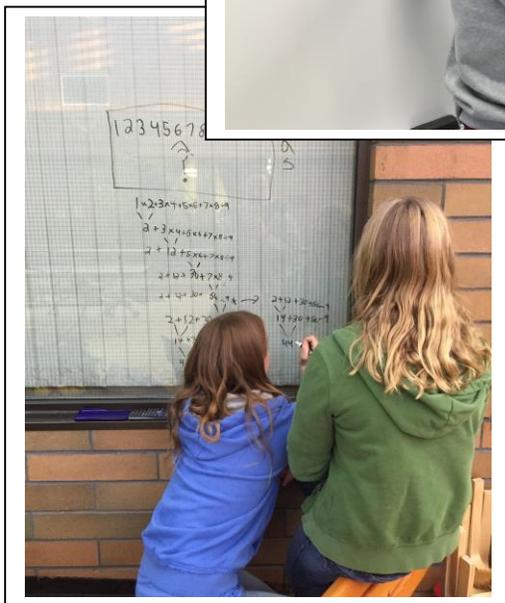
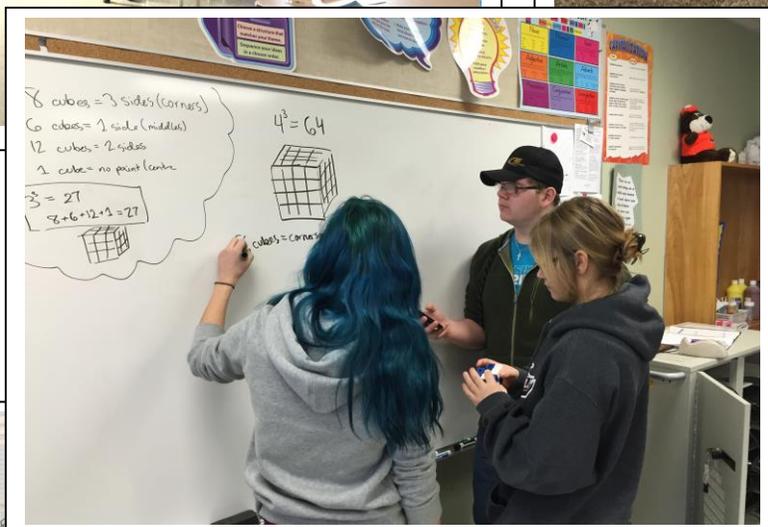
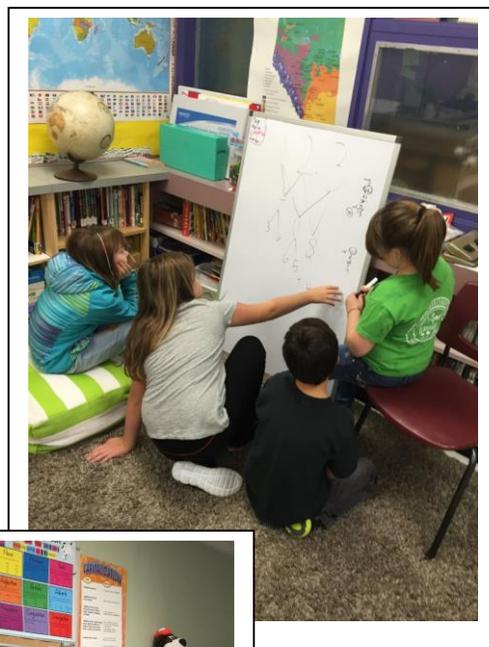
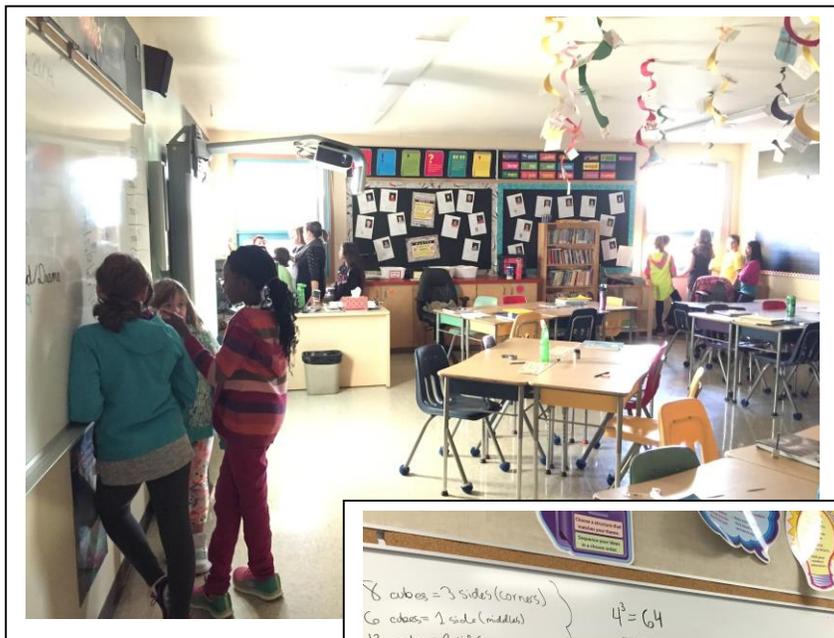


***“Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers. Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type How would you ...? or How could you ...?, the problem-solving approach is being modeled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies” (Alberta Mathematics Program of Studies, 2007).***

Teaching through problem solving is about inviting students to think about mathematics, to take risks, and to persevere. Collaboration is the key component of problem solving. Students need to be working together, sharing strategies, and learning from one another. The role of the teacher is to inspire, facilitate, and regulate. No telling, no showing, no giving answers. Your job is to motivate, question, and direct attention to big ideas!

Problem solving is our focus and problem solving is our lesson. This collection includes low- floor, high-ceiling problems with multiple entry points enabling all students to access and experience success with the problems. In our experience, teaching through problem solving levels the playing field. Students will struggle; this struggle will help them deepen their understanding and expand their skills. Problem solving gives the chance for all learners to be creative, think outside the box, and have a voice.

***“Coming to know something is not a spectator sport although numerous textbooks, especially in mathematics, and traditional modes of instruction may give that impression” (Brown and Walter, The Art of Problem Posing).***

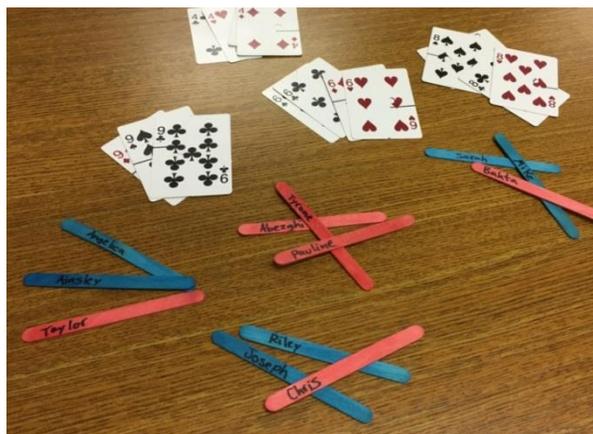


## Getting Started With Students

### **Random Groups + Non-Permanent Pen + Vertical Surfaces + Group Work, Collaboration, Communication + Different Skills and Strategies = A Thinking Classroom**

Students should work in **random groups**. This can be done using Popsicle sticks, a deck of cards, the random group generator on the Smart Board, etc. This will teach students how to work with everybody and anybody. This helps break down social barriers and nurtures a learning community in which students feel safe to take risks and make mistakes. This also helps prevent students from being labeled and grouped based on their “pre-conceived” mathematical abilities. If it is always random, it is always fair; the students know that the groups will always change and that they are expected to be able to work with everybody.

*(For more information, please read **THE AFFORDANCES OF USING VISIBLY RANDOM GROUPS IN A MATHEMATICS CLASSROOM** by Peter Liljedahl, Simon Fraser University, Canada – In press)*



Students should work at **vertical surfaces**. This allows everyone to have access to the workspace. It also allows for the teacher to easily see how each group is working, and who needs some direction, motivation, or extra help. Vertical surfaces are easily accessible by teachers for formative assessment. By standing in the middle of the room, it is possible to see where everybody is at. It allows both students and teachers to see at-a-glance the problem solving process, identify misconceptions, direct questioning, redirect the students, motivate group work, plan for discussions, mini-lessons and future lessons. Students' initial work should be on a **non-permanent** surface which encourages the risk-taking necessary for true problem solving. The non-permanence of the surface allows students to make mistakes without any long term consequences.

Whiteboards, windows, lockers, filing cabinets, shower curtains, shelf liner, writeable paint, table and desktops, and interactive whiteboards are a few examples of non-permanent vertical surfaces. Be sure to check the surface to ensure that the dry erase marker comes off prior to students writing on it.



Students need to develop and practice **group work, collaboration, and communication skills**. They need to learn how to listen to each other, to share their ideas, to question, and to trust their abilities and the abilities of others. **Different skills and strategies** need to be embraced while helping each other to create a safe learning environment.

*(For more information, please read **BUILDING THINKING CLASSROOMS: CONDITIONS FOR PROBLEM SOLVING** by Peter Liljedahl, Simon Fraser University, Canada – In press)*

**Big Marker Video, Building Thinking Classrooms, <https://www.youtube.com/watch?v=hc0hp0d15-4>**

## **Suggestions for Teaching Through Problem Solving:**

**Group sizes** depend on the teacher, the students, and the specific problem. We like 3 as a rule, but often have groups of 4 and occasionally students work in partners.

Students solve their problems in random groups at a vertical surface. There is only **one pen per group and it must be shared**. The person with the pen is not allowed to write down his or her ideas. Remind them not to hog the pen! This helps keep the groups working together.

**Gallery Walks / Mobilization of knowledge:** encourage the students to walk around the classroom to see other groups for ideas, to see different strategies, to get unstuck. This is also a great way to provide feedback instigate new discussions, and direct your teaching.

When you want to utilize a specific group's work to discuss a strategy, some specific math, misconceptions, etc., first **move all of the students to the center of the room, away from the work** in order to remove ownership of it (alleviate fear, embarrassment, etc.). Then move students back to the work to discuss it.

Encourage students to work together to work through the problem and get an answer. Tell them to work with their answer to see if they can find a more elegant way, to use a different strategy, to explain their ideas, and to **present their solution**.

**Use non-traditional assessments** such as observations, checklists, posters, videos, photos of work, written solutions that tell the story of how the problem was solved, etc. This can be done individually or in partners or groups. This allows students to show their problem solving process, to explain their thinking, and to showcase their understanding. Students can "present" their solutions as a group, in partners, or individually depending on what the teacher is assessing or needs to see.

Remember that problem solving takes practice. The more "traditional" learners may struggle to communicate and collaborate. It may take practice listening to other people's ideas and strategies. It can be frustrating working in groups and some students may find it difficult to explain their ideas. Many students lack confidence in math as well as in problem solving. **Students need to be taught how to think, how to collaborate, how to communicate, how to problem solve, and how to persevere.**

## **What is a Problem Worth Solving?**

A problem worth solving is accessible to all students. It has multiple entry points, has a low floor, wide walls, and a high ceiling. These problems lend themselves to natural differentiation where all students are able to address the problem at their level and experience success. A problem worth solving allows the use of multiple strategies and varying facets of mathematics.

“A problem-solving activity must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement”.

(Alberta Mathematics Program of Studies, 2007).

The **Painted Cube Problem is an exemplary problem** and it is our favourite example of a problem worth solving.

# The Painted Cube

**N1 PR2 SS3 SS4**

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*\*Credit to David Pimm*

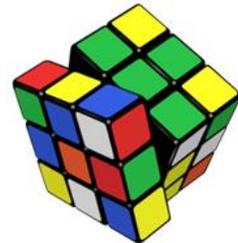
## Problem:

Picture a Rubik's Cube. Now drop it into paint so that it is completely covered. When the paint is dry, imagine smashing it on the floor and it breaking it apart into the smaller cubes.

How many of the cubes have one face covered in paint? How many cubes have two faces covered in paint? How many have three faces covered in paint? How many have zero faces covered in paint?

How could you predict the above for any size Rubik's cube?

What about a  $4 \times 4 \times 4$ ?  $5 \times 5 \times 5$ ?  $6 \times 6 \times 6$ ?  $N \times N \times N$ ?



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## Extension:

What if it wasn't a cube?

Why is the "one by one by one" cube a special case?

# The Painted Cube

## N1 PR2 SS3 SS4

### Outcome Objectives:

Number 1- Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

$$\bullet ax = b \bullet \frac{x}{a} = b, a \neq 0 \bullet ax + b = c \bullet \frac{x}{a} + b = c, a \neq 0 \bullet a(x + b) = c$$

where  $a$ ,  $b$  and  $c$  are integers. [C, CN, PS, V]

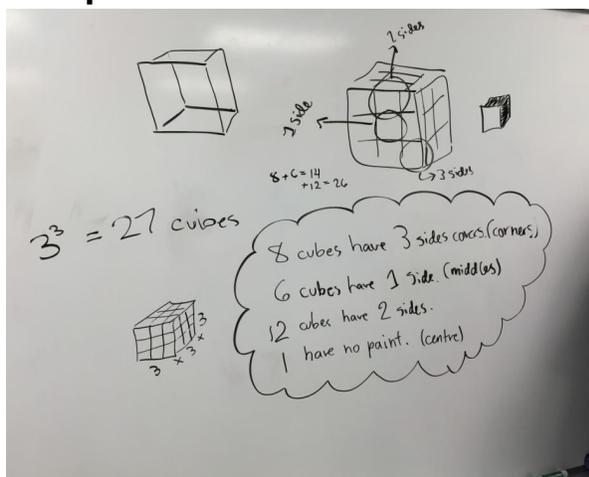
Shape and Space (3-D Objects and 2-D Shapes) 3- Determine the surface area of:   
 • right rectangular prisms • right triangular prisms • right cylinders to solve problems. [C, CN, PS, R, V]

Shape and Space (3-D Objects and 2-D Shapes) 4- Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders.

### Material Suggestions:

- Snap cubes
- Vertical surfaces
- Dry-erase markers

### Sample Solutions:



# The Painted Cube

# N1 PR2 SS3 SS4

## Student's Written Solution:

$D: (n-2)^3$   
 $L: (n-2)^2 \times 6$   
 $E: (n-2) \times 12$   
 $C: 8$  (always)

Examples (to see if they work):  
 To find how many cubes have certain amounts of painted sides you have to use the expressions above. I'll do this example with a  $19 \times 19 \times 19$  cube. All the dimensions of this cube are 19 by cubes large so in this case  $n=19$ .

0 sides painted:  $n=19$  so we take off two from 19 to get the length/width of the center of the square. After you must do 17 cubed because the little cubes that have 0 faces painted make a cube at the center of the big cube.

$19$   
 $- 2$   
 $17$

$17^3 = 4913$   
 $(17 \times 17 \times 17 = 4913)$

And 4913 would be your answer. There would be 4913 un-painted cubes.

1 side painted:  $n=19$  so you take 2 off of 19 again. 17 is the length of one edge (not including the corners) but the cubes with only 1 side painted are the cubes one row into the square/side. They make up the  $17 \times 17$  square on the side of the big cube so you have to do 17 squared ( $17 \times 17$ ).

$19$   
 $- 2$   
 $17$

$17^2 = 289$  ( $17 \times 17 = 289$ ). Now you have the number of cubes with 1 side painted on one side of the big cube but there's six sides on a cube so you have to multiply 289 by 6.  $289 \times 6 = 1734$ . So your answer is that there would be 1734 cubes with only one side painted.

Expression used:  $(n-2)^2 \times 6$



## A Visual Representation of a Solution:



## Mathematics related to the coloring of the cubes will emerge:

- Cubes with 3 faces painted: 8

These are always in the corners and there are always 8 (except on a size 1 cube).

- Cubes with 2 faces painted:  $12(n-2)$

These are always along the edges but not on the corners, so on each edge, there are 2 less than the size of the cube. There are 12 edges, so the number needs to be multiplied by 12.

- Cubes with 1 face painted:  $6(n-2)^2$

These are in the middle of each face. They are in the shape of a square, two sizes smaller than the face of the original cube. There are 6 faces, so the number needs to be multiplied by 6.

- Cubes with no faces painted:  $(n-2)^3$

These are always in the middle. They form a cube shape that is two sizes smaller than the original cube.

# The Painted Cube

**N1 PR2 SS3 SS4**

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## Notes:

This problem not only illustrates linear relationships, but also introduces and reinforces the idea that algebraic relations come from real situations and that can and should be visualized. Students can also graph the different relations.

Assessment idea: Have students initially solve the problem within a group, but individually write up their own solution which explain the process, tells the story of how the problem was solved, and explains the mathematics!

# How to Use This Resource

## *Section 1: Pages 16-61*

### [Problems to Create a Thinking Classroom \(Problems to Target the Front Matter\)](#)

The **first section** of problems is meant to be used to create a **thinking classroom**. Use these problems to teach students how to solve problems. These problems are included to address the Front-Matter of the curriculum as well as previous math concepts and outcomes. Students will go through the processes of learning to communicate, collaborate, reason, visualize, take risks, and persevere. The math classroom should become a culture of respect, responsibility, and thinking. Continue using these problems throughout the year!

**\*No sample solutions or curriculum links are provided for these problems. We want the teachers to learn with the students, take risks, make mistakes, and persevere! Feel free to google answers as a last resort if you really get stuck.**

## *Section 2: Pages 63-160*

### [Problems to Target Curricular Outcomes \(as Well as the Front Matter\)](#)

The **second section** of problems continues to build on these skills. These problems are **linked to the specific outcomes** in the grade 8 curriculum. As you are planning your lessons, select the problems in section two to best support your practice and to satisfy the needs of your students.

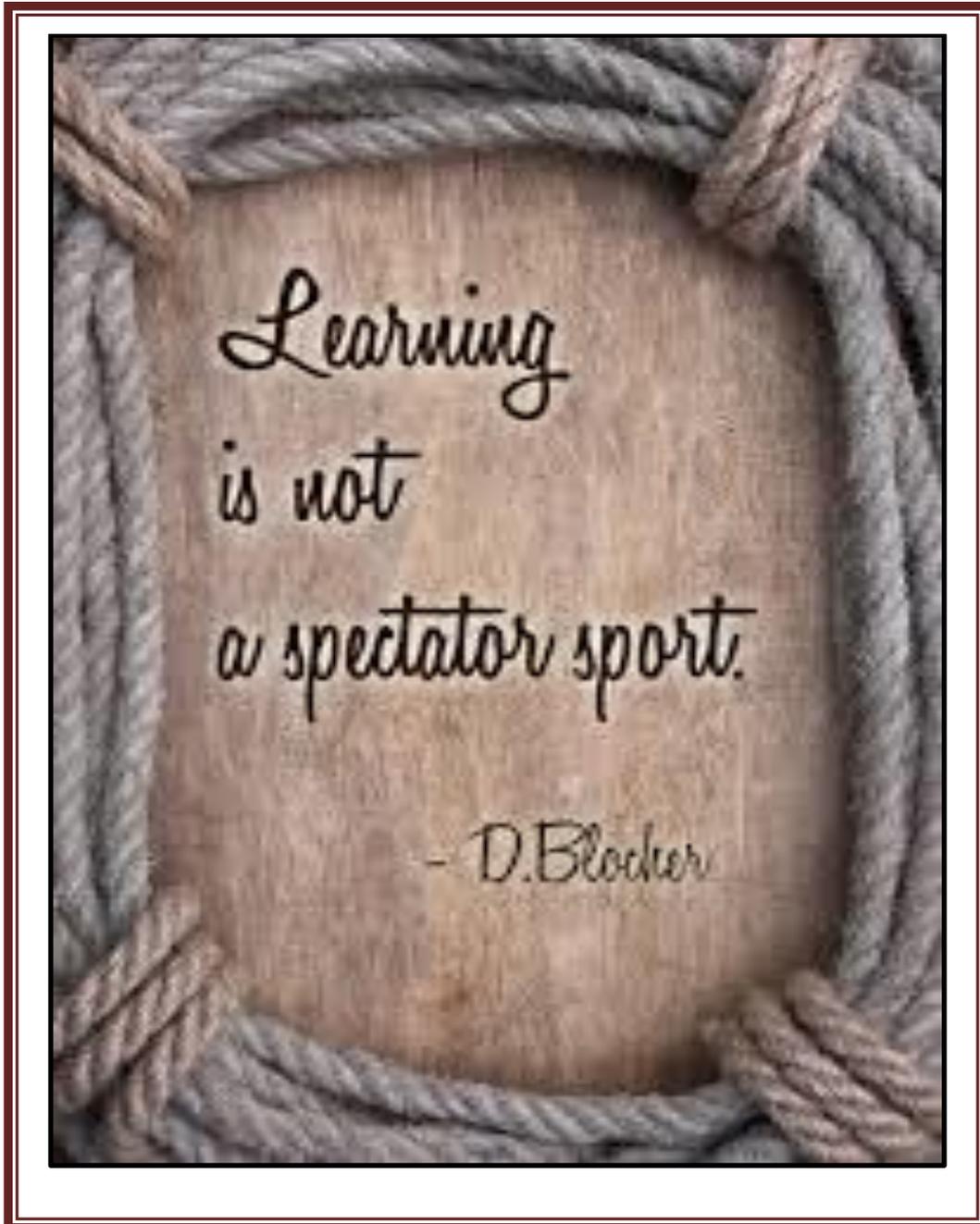
## *Section 3: Pages 162-210*

### [New Problems to Try Out in Your Classroom](#)

The **third section** of problems is a collection of new problems to experiment with in your Thinking Classroom. Some haven't been tried out yet. Some may not link to curriculum. Have fun and take a risk! Let us know how it goes.

*As you work your way through the junior high math curriculum by teaching through problem solving, please contact us with feedback, new ideas, exemplary problems, sample student solutions, etc. at [stone\\_alicia44@hotmail.com](mailto:stone_alicia44@hotmail.com).*

## Problems to Create a Thinking Classroom



**(Problems to Target the Front Matter)**

## Problems to Create a Thinking Classroom

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## 3 Questions

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*\*Credit to The Journal of Economic Perspectives, Vol. 19, No. 4 (Autumn, 2005), pp. 25-42*

### Problem:

**In a group, solve these 3 problems:**

**(1)** A bat and a ball cost \$1.10 in total. The bat costs \$1.00 more than the ball. How much does the ball cost?

**(2)** If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?

**(3)** In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?

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### Extension:

Can you create another problem in which your intuition may give you a wrong answer?

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### Notes:

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## Bell Boy and the Missing Dollar

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This is a 150 year-old problem!

### **Problem:**

Three men check into a hotel. The cost of the room is \$30. They pay \$10 each and check in. The manager discovers that he overcharged them because the cost of the room is actually only \$25. The manager gives the bellboy 5 loonies and tells him to return the money to the men. The Bell boy puts two loonies in his pocket and gives the men back one dollar each. Now each man has paid \$9 and the bellboy has two. That adds up to \$29. Where is the missing dollar?

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### **Extension:**

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### **Notes:**

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# Make 100

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*\*Credit to Ian Stewart*

## Problem:

Given the digits 1-9, make 100 using standard arithmetical symbols.

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## Extension:

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## Notes:

<http://www.quora.com/Mathematical-Puzzles/Can-you-make-100-out-of-the-digits-1-2-3-4-5-6-7-8-9-in-order>

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# One to One Hundred

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*\*Credit to Wilbert Reimer, Historical Connections in Mathematics, Vol 1*

## Problem:

Johann Friederich Carl Gauss was a mathematician born in Germany, on April 30, 1777. His parents were poor, and his father expected him to become a bricklayer or a gardener in the family tradition. Were it not for his strong mother and a persistent uncle, Gauss might not have received an education. In college, Gauss studied many subjects, but committed himself to mathematics as a lifelong pursuit. He was not interested in fame or wealth, but studied and explored primarily for personal satisfaction. For nearly 50 years, Gauss was professor of mathematics and astronomy and director of the observatory at the University of Göttingen. The “Prince of Mathematicians,” as Gauss was sometimes called, died February 23, 1855, at age 78. With Archimedes and Newton, he is considered one of the three greatest mathematicians who ever lived.

When Gauss was about 10, his teacher angrily assigned the class of boys a long problem: they should add the first one hundred numbers. Gauss was finished with the problem in a matter of seconds. The teacher was outraged. When the slates were checked, Gauss’s sum was correct.

How did Gauss solve this problem?

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## Extension:

Could you find the sum of the 50 first odd numbers?

How about the average of the first two even numbers? The first three even numbers? The first four even numbers? The first 50 even numbers?

The average of  $n$  even numbers?

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## Notes:

[BACK](#)

## Bees in the Trees

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*\*Credit to The Super Source: Patterns and Functions, grades 7-8*

### **Problem:**

The family tree of a male bee is both unusual and interesting. The male bee is created through a process known as parthenogenesis, whereby he has a single parent, only a mother: The female bee, however; has both a mother and a father: What patterns can you find by tracing the ancestry of a male bee?

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### **Extension:**

What if...you examined the ancestry of a species in which both males and females have two parents? How would the patterns be different from those you found with the bumblebees?

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### **Notes:**

[BACK](#)

# Palindromes

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*\*Credit to Peter Liljedahl*

## Problem:

A palindrome is a number, word, phrase, or sequence that reads the same backward as forward, e.g., madam or 363

Consider a two-digit number – for example 84. 84 is not a palindrome. So, reverse the digits and add it to the original number –  $84 + 48 = 132$ . Repeat this process until the sum becomes a palindrome.  $132 + 231 = 363$ . The number of times the process is repeated determines the depth of the palindrome. For 84, the depth is two. Find the depth of all two digit numbers.

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## Extension:

What about a three digit number?

What about the depth for the second time of becoming a palindrome?

What happens when you shade a Hundreds Chart according to the number's depth?

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## Notes:

<http://www.magic-squares.net/palindromes.htm>

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# Marching Band

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*\*Credit to John Grant McLoughlin*

## **Problem:**

Students in a marching band want to line up for their performance. The problem is that when they line up in twos there is 1 left over. When they line up in threes there are 2 left over. When they line up in fours there are 3 left over. When they line up in fives there are 4 left over. When they line up in sixes there are 5 left over. When they line up in sevens there are no students left over. How many students are there?

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## **Extension:**

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## **Notes:**

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# Fifteen

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*\*Credit to Peter Liljedahl and John Mason*

## Problem:

Using the numbers 1 2 3 4 5 6 7 8 9

Alternate between partners to pick one number at a time.

Once a number is picked, it is gone.

The goal is to have 3 numbers that add to 15.

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## Extension:

What are some strategies for winning?

What childhood game does this connect with?

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## Notes:

Here is a variation with manipulatives: Nine counters marked with the digits 1 to 9 are placed on the table. Two players alternately take one counter from the table. The winner is the first player to obtain, amongst his or her counters, three with the sum of exactly 15.

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# 4 Fours

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*\*Credit to Jo Boaler*

## Problem:

Can you make the numbers 1 through 10 by using only 4 fours and any operations?

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## Extension:

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## Notes:

Visit <https://www.youcubed.org/tasks/> for more tasks.

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## How to Win at 21

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### **Problem:**

Play with a partner. You need 21 snap cubes or other objects. The goal is to make your partner take the last object.

Snap the 21 cubes together in a chain. The first player takes off 1, 2, or 3 cubes off the chain. Then your partner can take off 1, 2, or 3. The person who has to take the last cube loses.

Play multiple games. Figure out how to win the game!

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### **Extension:**

Play again but this time you can take 2, 3, or 4 cubes each turn.

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### **Notes:**

See You Tube video:

[https://youtu.be/XD\\_HRBugh34](https://youtu.be/XD_HRBugh34)

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## Frame the Cards

*\*Credit to Ian Stewart*

### Problem:

Arrange the cards from the ace to the ten into a picture frame so that each the top, bottom, and sides add to the same total of spots (hearts/diamonds...) Right now the top row adds to 23, the bottom adds to 12, the left side is 22 and the right side is 22. These four numbers should be the same. Apparently there are 10 solutions to this problem.



### Extension:

Can you build your own?

### Notes:

[BACK](#)

## 30 Scratch

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*\*Credit to John Grant McLoughlin*

### Problem:

Roll a die to choose 4 digits from 2-9 e.g. 3 5 7 9

Use these digits in combination with any operation to make the numbers 1-30.

There are three basic "not allowed rules":

- Digits cannot be used twice. So  $3 \times 5 = 15$  and then minus 3 would not be acceptable for 12.
- Digits cannot be put together to make two-digit numbers, so 3 and 5 cannot be used to write 35
- Only basic operations are allowed (no square roots, exponents, etc.)....**brackets are allowed.**

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### Extension:

What if we did allow other operations?

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### Notes:

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# Free Throw

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*\*Credit to Geri Lorway*

## Problem:

Bill, Sam and Rob all play for the school basketball team and are very consistent free throw shooters (although not necessarily consistently good).

Bill always misses one shot then makes one shot.

Sam always misses two shots then makes one shot.

Rob always misses three shots then makes one shot.

The three boys are practicing their free throws in rounds where each gets one shot per round. Predictably, all three boys miss in the first round and each of them stays on their usual pattern for the rest of the rounds.

After **16 rounds** they stop to compare the baskets they have made in each round.

What if they played more than 6 rounds? What about 57 rounds?

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## Extension:

What questions do you want to answer? Answer them!

Examples of Questions from Students:

In how many rounds did Bill make his shot?

How many times did both Sam and Rob make their shot in the same round?

How many times did all three players make shots in the same round?

How many times did all three players miss their shot in the same round?

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## Notes:

[BACK](#)

# Seed Numbers

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*\*Credit to Peter Liljedahl*

## Problem:

Consider the following pattern of 5 **whole** numbers, where each number is the sum of the previous two numbers:

3, 12, 15, 27, 42

I want the 5th number to be 100.

Find all the **whole seed numbers that will make this so (the seed numbers are the first two whole numbers)**.

---

## Extension:

---

## Notes:

[BACK](#)

# Egg Timer

---

*\*Credit to Rina Zazkis*

## Problem:

You have 2 hourglass egg timers, a 7-minute timer and a 4-minute timer, and you want to boil a 9-minute egg. How do you time it exactly? You can ignore the time it takes to reset the timers.

---

## Extension:

What if you had a 7-minute timer and an 11-minute timer, and wanted to cook a 15-minute egg?

---

## Notes:

Sample solution: <http://www.greylabyrinth.com/solution/puzzle013>

[BACK](#)

# 1001 Pennies

---

## Problem:

On a table, there are 1001 pennies lined up in a row. I then come along and replace every second coin with a nickel. After this, I replace every third coin with a dime. Finally, I replace every fourth coin with a quarter. After all this, how much money is on the table?

---

## Extension:

---

## Notes:

[BACK](#)

# Crossing the Bridge

---

*\*Credit to <http://puzzles.nigelcoldwell.co.uk/twentyfive.htm>*

## Problem:

Four people are being pursued by a menacing beast. It is nighttime, and they need to cross a bridge to reach safety. It is pitch black, and only two can cross at once. They need to carry a lamp to light their way. The first person can cross the bridge in 10 minutes, the second in 5 minutes, the third in 2 minutes, and the fourth in 1 minute. If two cross together, the couple is only as fast as the slowest person. (That is, a fast person can't carry a slower person to save time, for example. If the 10-minute person and the 1-minute person cross the bridge together, it will take them 10 minutes.) The person or couple crossing the bridge needs the lamp for the entire crossing and the lamp must be carried back and forth across the bridge (no throwing, etc.) How long will it take them to get across? Is this the shortest time possible?

---

## Extension:

*\*Credit to @BCAMT*

Another Bridge Problem: Five people are standing on one side of a bridge. They want to cross the bridge. Without a torch, they cannot proceed. Only one torch is available. The torch has a remaining battery life of only 30 seconds. Only two people can go over the bridge at one time. The torch needs to be returned to the remaining people. The five people take different times to cross the bridge. One takes 1 second to cross. The others take 3 seconds, 6 seconds, 8 seconds and 12 seconds. Everyone crosses the bridge within 30 seconds. How do they proceed?

What if the time taken for two to cross is the average of their two times?

---

## Notes:

Here is a video of the problem: <http://ed.ted.com/lessons/can-you-solve-the-bridge-riddle-alex-gendler>

[BACK](#)

# Spell the Cards

---

*\*Credit to Peter Liljedahl*

## Problem:

Rig the cards (Ace to King) so that you spell the cards (see YouTube video).

<http://www.youtube.com/watch?v=qDgcLAGgX2A>

Order of cards	Spelling
1.	Ace
2.	Two
3.	Three
4.	Four
5.	Five
6.	Six
7.	Seven
8.	Eight
9.	Nine
10.	Ten
11.	Jack
12.	Queen
13.	King

---

**Extension:** Could you rig the whole deck? What about in another language?

---

## Notes:**\*for the teacher only\***

Rig the deck first and perform the card trick for your students. Ask “how”.

Queen, Four, Ace, Eight, King, Two, Seven, Five, Ten, Jack, Three, Six, Nine

[BACK](#)

# The Silver Bar

---

*\*Credit to Martin Gardner*

## Problem:

A silver prospector was unable to pay his March rent in advance. He owned a bar of pure silver, 31 inches long, so he made the following arrangement with his landlady. He would cut the bar, he said, into smaller pieces and pay her in silver (one inch per day). On the first day of March he would give the lady an inch of the bar, and on each succeeding day he would add another inch to her amount of silver. He doesn't want to cut the bar into 31 pieces because it required considerable labor – he wished to carry out his agreement with the fewest possible number of pieces. Assuming that portions of the bar can be traded back and forth, what is the smallest number of pieces in which the prospector needs to cut his silver bar?

---

## Extension:

---

## Notes:

[BACK](#)

## Hoax (apparently this was a joke)

---

### Problem:

29  
AUG

### Samsung pays Apple \$1 Billion sending 30 trucks full of 5 cent coins

PaperBlog – This morning more than 30 trucks filled with 5-cent coins arrived at [Apple's](#) headquarters in California. Initially, the security company that protects the facility said the trucks were in the wrong place, but minutes later, [Tim Cook \(Apple CEO\)](#) received a call from [Samsung](#) CEO explaining that they will pay \$1 billion dollars for the fine recently ruled against the South Korean company in this way.



The [funny](#) part is that the signed document does not specify a single payment method, so Samsung is entitled to send the creators of the [iPhone](#) their billion dollars in the way they deem best.

This dirty but genius geek [troll](#) play is a new headache to Apple executives as they will need to put in long hours counting all that [money](#), to check if it is all there and to try to deposit it crossing fingers to hope a bank will accept all the coins.

What questions do you have? How would you respond if you were Apple?

---

### Extension:

What about if they delivered 1 Billion in dimes?

---

### Notes:

[BACK](#)

# The Gold Chain

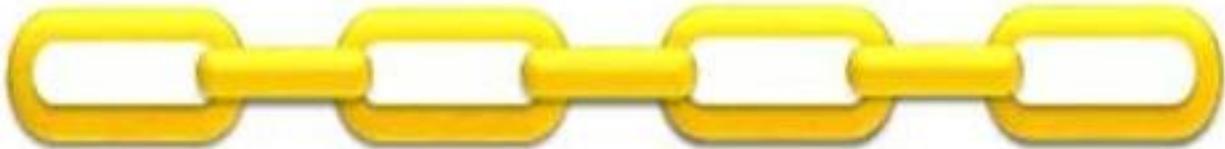
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\*Credit to [TestFunda](#) vol. 1 (p. 1)

## Problem:

A wealthy man needed to pay the mason building his house. He was running low on cash so he decided to pay the mason with a gold chain made of 7 links. The mason's fee was equal to one gold link each day. The wealthy man needed to pay the mason each day or he would stop working. But, at the same time he didn't want to pay the mason any more than one link in a day because he might run off and not return.

Cutting the chain was very difficult. What is the minimum number of cuts that the wealthy man could make in the chain and still pay the mason each day for seven days?



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## Extension:

---

## Notes:

[BACK](#)

## Same Sum

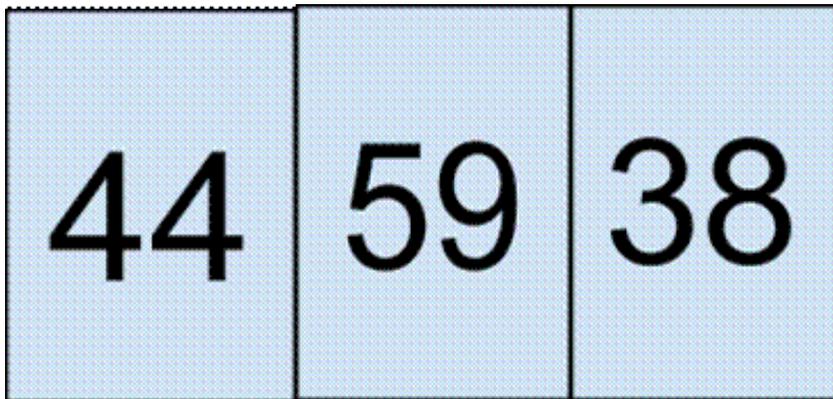
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*\*Credit to Rina Zazkis*

### Problem:

You have three cards in front of you. On the back of each of the cards is a different prime number. The sum of the number on the front and the number on the back is the same for each card.

-What are the prime numbers on the back of the cards?



---

### Extension:

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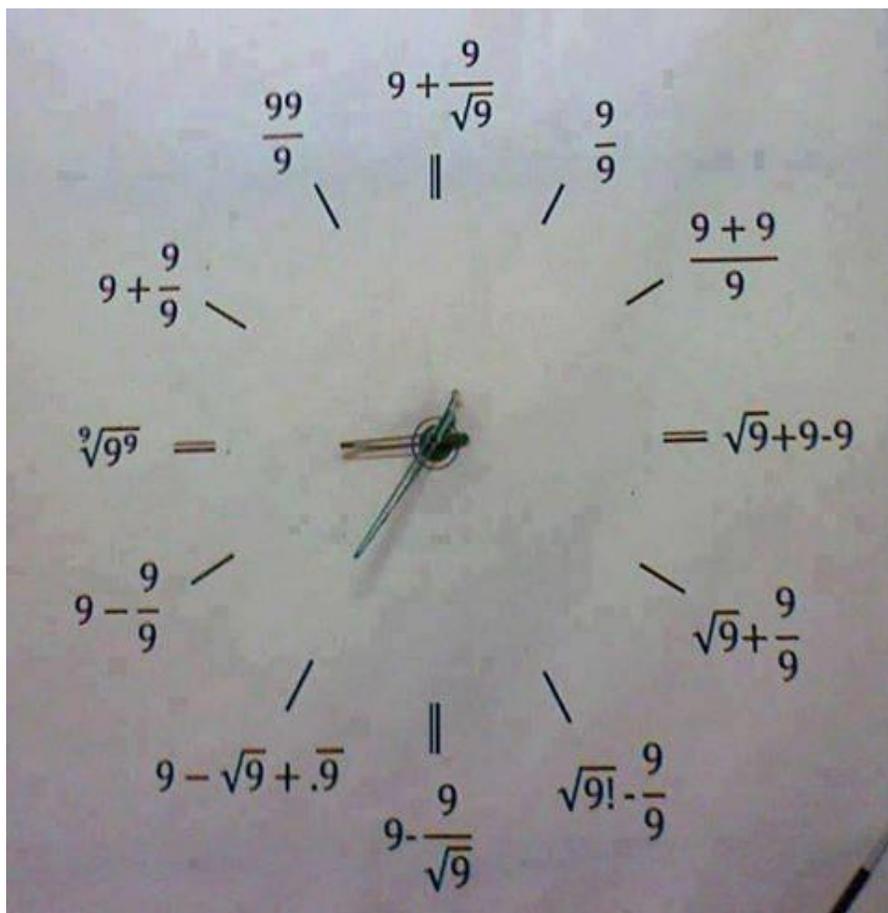
### Notes:

[BACK](#)

## Square Root Clock

---

Problem:



Will this clock tell the right time? Prove it mathematically!

---

Extension:

---

Notes:

[BACK](#)

# Triangular Numbers

---

## Problem:

The numbers 1,3,6,10,15,... are known as triangular numbers.



Why?

How do you find more?

How do you know if any number is a triangular number?

---

## Extension:

What is the largest triangular number less than 500?

---

## Notes:

[BACK](#)

# Pirate Diamond

---

*\*Credit to Peter Liljedahl*

## Problem:

A band of 10 pirates are going to disband. They have divided up all of their gold, but there remains one GIANT diamond that cannot be divided. To decide who gets it the captain puts all of the pirates (including himself) in a circle. Then he points at one person to begin. This person steps out of the circle, takes his gold, and leaves. The person on his left stays in the circle, but the next person steps out. This continues with every second pirate leaving until there is only one left. Who should the captain point at if he wants to make sure he gets to keep the diamond for himself?

---

## Extension:

What if there were 11 pirates? What if there were 12 pirates? What if there were 27 pirates?

---

## Notes:

[BACK](#)

# Mother-Daughter Tea Party

---

*\*Credit to Rina Zazkis*

## Problem:

There is a party for mothers and daughters. All the mothers shake hands among themselves (but not the daughters). Every daughter shakes hands with all the mothers. How many handshakes are there, if it is known that the party involved 17 mother-daughter pairs?

---

## Extension:

What if there were more mother-daughter pairs?

What if every mother had two daughters?

---

## Notes:

[BACK](#)

# Rabbits

---

## Problem:

Suppose a one month old pair of rabbits (one male and one female) are too young to reproduce, but are mature enough to reproduce when they are two months old. Also assume that every month, starting from the second month, they produce a new pair of rabbits (one male, one female).

If each pair of rabbits reproduces in the same way as the above, how many pairs of rabbits will there be at the beginning of each month?

How many pairs will there be after one year?

---

## Extension:

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## Notes:

[BACK](#)

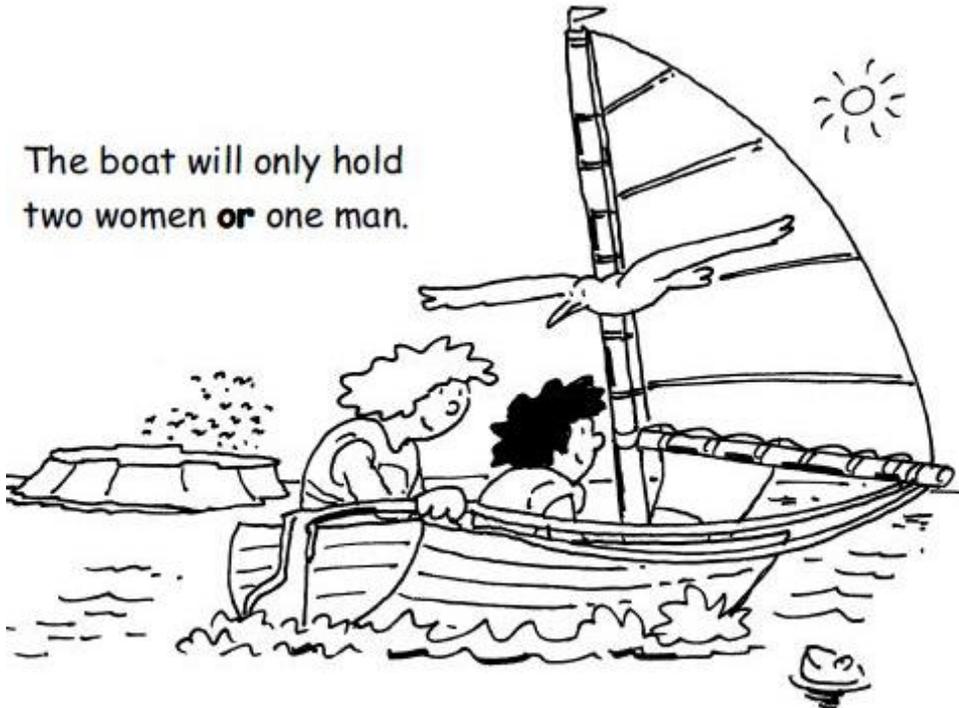
# Sail Away

*\*Credit to Mathematical Challenges for Able Pupils*

## Problem:

Two men and two women want to sail to an island.

The boat will only hold two women **or** one man.



How can all four of them get to the island?

How many trips will it take?

---

## Extension:

---

## Notes:

[BACK](#)

## Jugs – Die Hard 3

---

*\*Credit to the movie Die Hard With a Vengeance*

### **Problem:**

You have a three gallon jug and a five gallon jug. You need to measure out exactly 4 gallons. How can you do this? Is there more than one way?

---

### **Extension:**

Make 1 gallon from a 5 gallon jug and a 7 gallon jug

Make 6 gallons from a 4 gallon jug and a 9 gallon jug

What other amounts can you make from different sized jugs?

---

### **Notes:**

Search for a YouTube Clip of the scene in the movie (be careful of language).

## How Many Heartbeats?

---

### **Problem:**

How long does it take for your heart to beat 1000 times? If you started counting at midnight tonight, when would you count the millionth beat? What about the billionth beat?

---

### **Extension:**

When was a million seconds ago? When was a billion minutes ago? When was a billion hours ago?

---

### **Notes:**

[BACK](#)

## Paper to the Moon

---

### **Problem:**

Imagine you have a really long piece of paper and you folded it in half (doubling its thickness, and then in half again (doubling it again), and then in half again (and so forth), how many folds would it take so that the total thickness of your paper could reach the moon?

---

### **Extension:**

---

### **Notes:**

## 21 Casks

---

*\*Credit to Malba Tahan, The Man Who Counted*

### **Problem:**

The sheik addressed the three of them: “Here are my three friends. They are sheep rearers from Damascus. They are facing one of the strangest problems I have come across. It is this: as payment for a small flock of sheep, they received, here in Baghdad, a quantity of excellent wine, in 21 identical casks: 7 full, 7 half full, 7 empty. They now want to divide these casks so that each receives the same number of casks and the same quantity of wine. Dividing up the casks is easy – each would receive 7. The difficulty, as I understand it, is in dividing the wine without opening them, leaving them just as they are. Now, calculator, is it possible to find a satisfactory answer to this problem?

**-The Man Who Counted**

---

### **Extension:**

---

### **Notes:**

[BACK](#)

## Diagonals in a Polygon

---

### Problem:

If given the number of sides in a polygon, can you determine the number of diagonals?

---

### Extension:

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### Notes:

[BACK](#)

## More Fractions

---

*\*First two credit to Geri Lorway, second two credit to NCTM*

### Problems:

#### Pizza Problem

At Pizza Hut, 14 girls equally shared 6 large pizzas and 6 boys equally shared 2 large pizzas. Who got to eat more pizza, a boy or a girl?

#### Bottles of Pop

At a party 5 girls equally shared 3 bottles of Pop and 6 boys equally shared 2 bottles of Pop. The bottles were the same size. Who would get to drink more pop? The boys or the girls?

#### I am a proper fraction.

The sum of my numerator and denominator is 1 less than a perfect square. Their difference is 1 more than a perfect square. Their product is 1 less than a perfect square. Who am I?

**A half is a third of a fourth. What is it?**

---

### Extension:

---

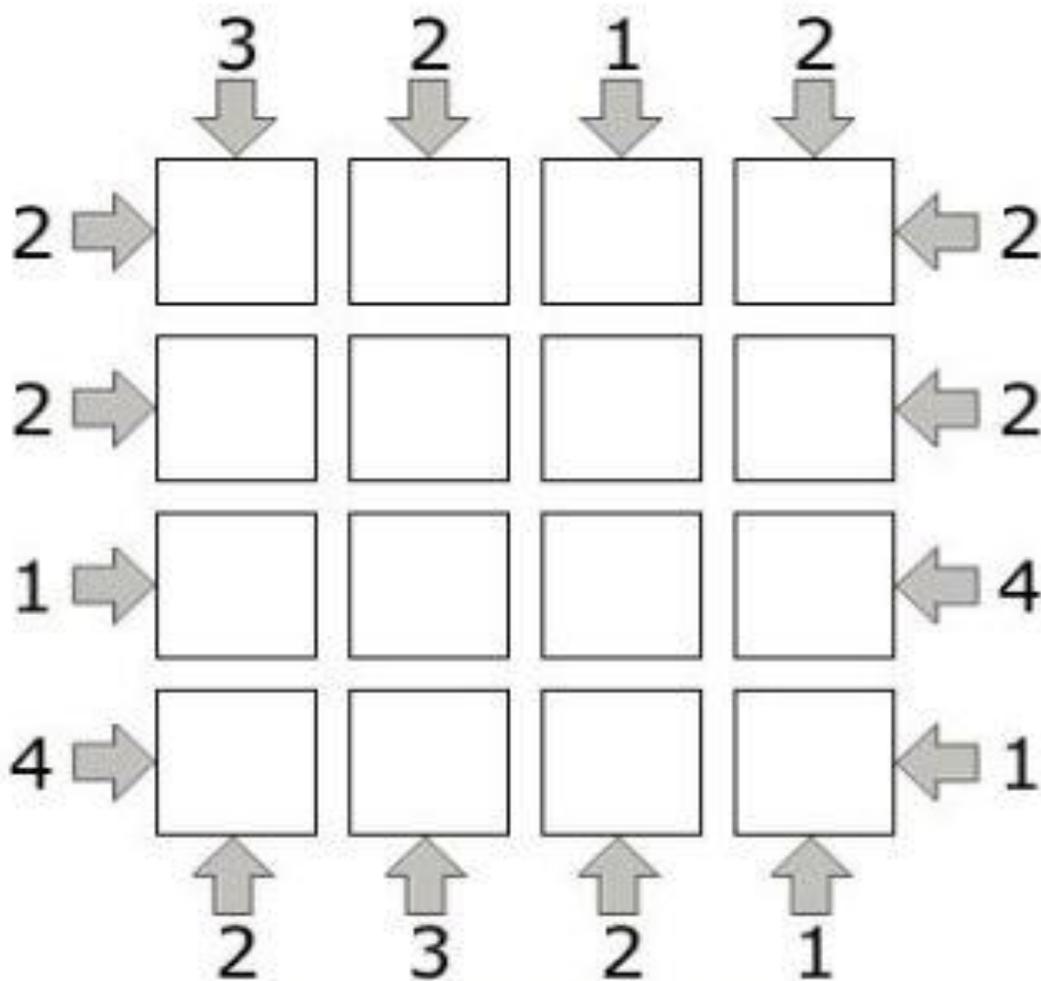
### Notes:

[BACK](#)

# Skyscrapers

\*Credit to Kevin Stone at [www.brainbashers.com](http://www.brainbashers.com)

**Problem:**



Skyscraper Puzzle © Kevin Stone

Complete the grid such that every row and column contains the numbers one to four.

Each row and column contains each number only once.

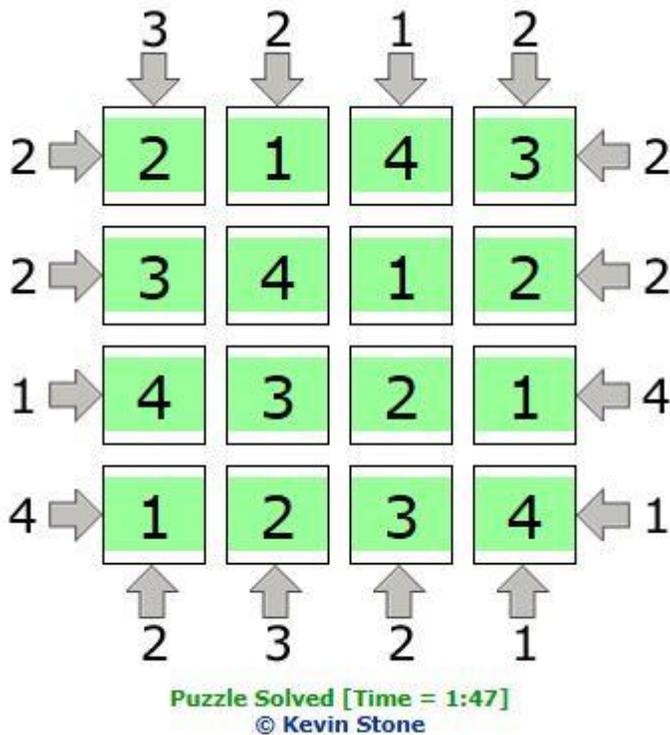
The numbers around the outside tell you how many skyscrapers you can see from that view.

You can't see a shorter skyscraper behind a taller one.

# Skyscrapers continued

Extension:

Notes:



Visit <https://www.brainbashers.com/showskyscraper.asp?date=0508&size=9&diff=1>

Print out puzzle for students (enlarge it so cubes fit).

Provide snap cubes to build towers.

[BACK](#)

# Rope Around the World

---

*\*Credit to Rina Zazkis*

## Problem:

If we wanted to hold a rope around the world, how much rope would we need? If we held the rope 1m higher, how much more rope would we need?

Variation: If my rope was 10m longer than the circumference of the world, could an ant fit under it? What about a mouse, a dog, or a human?

---

## Extension:

What if the world was a cube instead of a sphere?

---

## Notes:

[BACK](#)

# Ten Divisors

---

*\*Credit to Derrick Niederman*

## Problem:

What is the smallest positive number with exactly ten positive integer divisors?

And what is the next one after that?

---

## Extension:

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## Notes:

[BACK](#)

# Committees

---

**Problem:**

How many committees of 3 people can be formed from a group of 12 people?

---

**Extension:**

What if there were more total people? What if there were more people on the committees?

---

**Notes:**

# Dollar

---

**Problem:**

How many different ways are there to make a dollar using quarter, dimes, and nickels?

---

**Extension:**

How about two dollars?

What if you added pennies?

---

**Notes:**

[BACK](#)

## Line Them Up

---

### Problem:

A preschool class has 6 students. Every day they line up to go to lunch. They like to line up in a different order each time. After how many days will it no longer be possible to choose an order that they have not used before?

---

### Extension:

What if there were more students?

---

### Notes:

## Desk Calendar

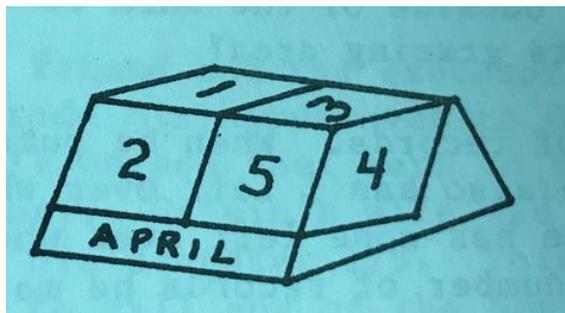
---

### Problem:

In a doctor's office, there is a desk calendar made of 2 cubes to show the day of the month (1-31). What digits have to be on each cube so that every day of any month can be represented properly?



### Extension:



On the desk calendar above, find the four digits that can't be seen on the left cube and the three digits that can't be seen on the right cube.

### Notes:

[BACK](#)

# Lacrosse Tournament

---

*\*Credit to Howard County Common Core Math Resources*

## Problem:

The Parks and Recreation Department is planning a tournament for club lacrosse teams in the area. Sixty-four teams have entered to play. Teams will be placed in a single elimination bracket at random. Four fields are available for tournament use. Games will only be played on Saturdays from 8:00 am until 4:00 pm. Games are 40 minutes in length. Ten minutes is allotted for half time and 10 minutes for teams to warm up. A team can play only one round each Saturday.

Your job is to report how many fields are being used each Saturday during the tournament. The parks department also needs to know how much available field space they have each weekend for other activities.

---

## Extension:

---

## Notes:

Visit <https://secondarymathcommoncore.wikispaces.gcpss.org/>

[BACK](#)

## All of the Primes

---

### Problem:

Find all of the prime numbers between 1-100.

---

### Extension:

What about between 100 and 200?

---

### Notes:

[BACK](#)

# Ball of Yarn

---

*\*Credit to David Pimm*

## Problem:

How do we pass a ball of yard around a circle so that everyone touches the ball? Start with a circle of an odd number of people.

Pass a ball of yarn around the circle skipping every 2nd person. Everyone has to touch the ball of yarn. How many times is the ball of yarn touched? What shape is made? How many intersections of yarn will be made?

What happens when the number of people in the circle changes? How can you make sure that everyone in the circle can touch the ball of yarn?

---

## Extension:

What happens when the passing interval changes? How can you be sure that everyone in the circle can touch the ball of yarn?

---

## Notes:

Use a ball of yard. Everybody holds on to a section as they pass the ball to the next person. Take time with this one!!!

[BACK](#)

## Problems to Target Curricular Outcomes



I FACILITATE THINKING

*I engage minds*

I LISTEN TO QUESTIONS

I encourage risk

I support struggle

*I cultivate dreams*

I LEARN EVERY DAY

I TEACH



As Well as the Front Matter

<b>Problem</b>	<b>Pg.</b>	<b>N1</b>	<b>N2</b>	<b>N3</b>	<b>N4</b>	<b>N5</b>	<b>N6</b>	<b>N7</b>	<b>PR1</b>	<b>PR2</b>
<a href="#">Painted Cube</a>	65	X								X
<a href="#">Checkerboard</a>	69	X								
<a href="#">Open Lockers</a>	71	X								
<a href="#">How Many Dots?</a>	73	X							X	X
<a href="#">Discount Machines</a>	75			X						
<a href="#">Percent Benchmarks</a>	77			X						
<a href="#">Amazing Offer</a>	79				X	X				
<a href="#">Cell Phone Plans</a>	81				X	X				
<a href="#">The Jeweler</a>	83				X	X				
<a href="#">12 Cookies</a>	85				X	X				
<a href="#">Bathtub</a>	87				X	X	X			
<a href="#">Retirement</a>	89				X	X	X			
<a href="#">Bungee Barbie</a>	91				X	X			X	X
<a href="#">Grannie Stitchwork</a>	93				X	X				
<a href="#">Pennies</a>	95					X	X			
<a href="#">Traffic Lights</a>	97					X				
<a href="#">Bears and Apples</a>	99						X			
<a href="#">Birds on a Lake</a>	101						X			
<a href="#">Cookie Jar</a>	103						X			
<a href="#">Fractions/Decimals/Percents</a>	105				X		X			
<a href="#">Sharing Bread</a>	107						X			
<a href="#">Giving Out Bonuses</a>	109						X			
<a href="#">Sharing Sub Sandwiches</a>	111						X			
<a href="#">Integers Progression</a>	113							X		
<a href="#">Algebra Sort</a>	122							X		X
<a href="#">Moving Squares</a>	126								X	X
<a href="#">Growing Pattern</a>	128								X	X
<a href="#">Mountain Range</a>	130								X	X
<a href="#">Picture Frames</a>	132								X	X
<a href="#">Pool Border</a>	134								X	X
<a href="#">Staircase</a>	136								X	X
<a href="#">Toothpicks</a>	138								X	X
<a href="#">Skyscraper Windows</a>	140								X	X
<a href="#">51-Foot Ladder</a>	143									
<a href="#">Pringles</a>	145									
<a href="#">How much Popcorn?</a>	147									
<a href="#">Pentominoes</a>	149									
<a href="#">Eyewitness</a>	151									
<a href="#">Hat and Rabbit</a>	153									
<a href="#">The Monte Hall Problem</a>	155									
<a href="#">The Restaurant</a>	157					X				X
<a href="#">Cap and Umbrella</a>	159					X				X

SS1	SS2	SS3	SS4	SS5	SS6	SP1	SS2	Pg.	Problem
		X	X					65	<a href="#">Painted Cube</a>
								69	<a href="#">Checkerboard</a>
								71	<a href="#">Open Lockers</a>
								73	<a href="#">How Many Dots?</a>
								75	<a href="#">Discount Machines</a>
								77	<a href="#">Percent Benchmarks</a>
								79	<a href="#">Amazing Offer</a>
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								87	<a href="#">Bathtub</a>
								89	<a href="#">Retirement</a>
								91	<a href="#">Bungee Barbie</a>
X	X	X	X					93	<a href="#">Grannie Stitchwork</a>
								95	<a href="#">Pennies</a>
							X	97	<a href="#">Traffic Lights</a>
								99	<a href="#">Bears and Apples</a>
								101	<a href="#">Birds on a Lake</a>
								103	<a href="#">Cookie Jar</a>
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								138	<a href="#">Toothpicks</a>
	X							140	<a href="#">Skyscraper Windows</a>
X								143	<a href="#">51-Foot Ladder</a>
X		X						145	<a href="#">Pringles</a>
	X	X	X					147	<a href="#">How Much Popcorn?</a>
				X	X			149	<a href="#">Pentominoes</a>
							X	151	<a href="#">Eyewitness</a>
							X	153	<a href="#">Hat and Rabbit</a>
							X	155	<a href="#">The Monte Hall Problem</a>
								157	<a href="#">The Restaurant</a>
								159	<a href="#">Cap and Umbrella</a>

# The Painted Cube

**N1 PR2 SS3 SS4**

*\*Credit to David Pimm*

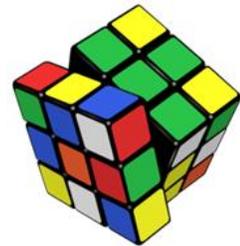
## Problem:

Picture a Rubik's Cube. Now drop it into paint so that it is completely covered. When the paint is dry, imagine smashing it on the floor and it breaking it apart into the smaller cubes.

How many of the cubes have one face covered in paint? How many cubes have two faces covered in paint? How many have three faces covered in paint? How many have zero faces covered in paint?

How could you predict the above for any size Rubik's cube?

What about a  $4 \times 4 \times 4$ ?  $5 \times 5 \times 5$ ?  $6 \times 6 \times 6$ ?  $N \times N \times N$ ?



---

## Extension:

Why is the "one by one by one" cube a special case?

What if it wasn't a cube?

[BACK](#)

# The Painted Cube

# N1 PR2 SS3 SS4

## Outcome Objectives:

Number 1- Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

$$\bullet ax = b \bullet \frac{x}{a} = b, a \neq 0 \bullet ax + b = c \bullet \frac{x}{a} + b = c, a \neq 0 \bullet a(x + b) = c$$

where  $a$ ,  $b$  and  $c$  are integers. [C, CN, PS, V]

Shape and Space (3-D Objects and 2-D Shapes) 3- Determine the surface area of:

• right rectangular prisms • right triangular prisms • right cylinders to solve problems.

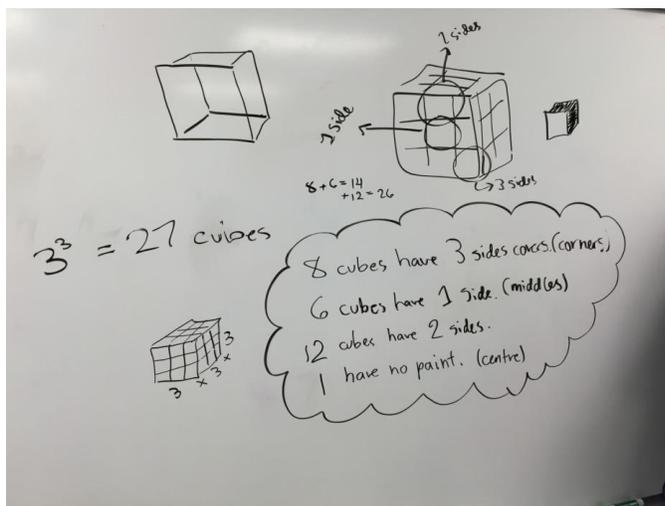
[C, CN, PS, R, V]

Shape and Space (3-D Objects and 2-D Shapes) 4- Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms and right cylinders.

## Material Suggestions:

- Snap cubes
- Vertical surfaces
- Dry-erase markers

## Sample Solutions: The beginnings of a solution:



# The Painted Cube

# N1 PR2 SS3 SS4

## Student's Written Solution:

$D: (n-2)^3$   
 $L: (n-2)^2 \times 6$   
 $E: (n-2) \times 12$   
 $S: 8$  (always)

Examples (to see if they work):  
 To find how many cubes have certain amounts of painted sides you have to use the expressions above. I'll do this example with a  $19 \times 19 \times 19$  cube. All the dimensions of this cube are 19 big cubes long so in this case  $n=19$ .

0 sides painted:  $n=19$  so we take off two from 19 to get the length/width of the center of the square. After, you must do  $17^3$  cubes because the 17 cubes that have 0 faces painted make a cube at the center of the big cube.

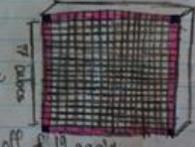
$17^3 = 4913$   
 $(17 \times 17 \times 17 = 4913)$

And 4913 would be your answer. There would be 4913 un-painted cubes.

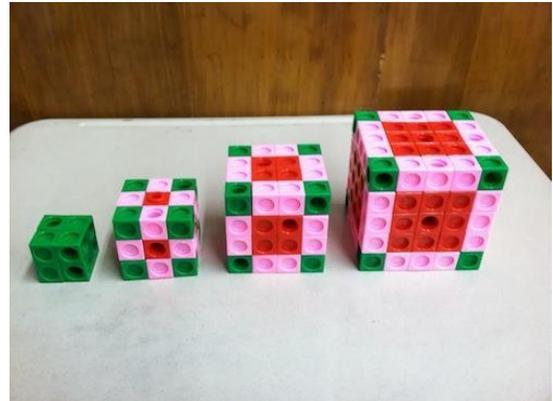
1 side painted:  $n=19$  so you take 2 off of 19 again. 17 is the length of one edge (not including the corners) but the cubes with only 1 side painted are the cubes one row into the square/side. They make up the  $17 \times 17$  square on the side of the big cube so you have to do  $17^2$  squared.

$17^2 = 289$  ( $17 \times 17 = 289$ ). Now you have the number of cubes with 1 side painted on one side of the big cube but there's six sides on a cube so you have to multiply 289 by 6.  $289 \times 6 = 1734$ . So your answer is that there would be 1734 cubes with only one side painted.

Expression used:  $(n-2)^2 \times 6$



## A Visual Representation of a Solution:



## Mathematics related to the coloring of the cubes will emerge:

- Cubes with 3 faces painted: 8  
These are always in the corners and there are always 8 (except on a size 1 cube).
- Cubes with 2 faces painted:  $12(n - 2)$   
These are always along the edges but not on the corners, so on each edge, there are 2 less than the size of the cube. There are 12 edges, so the number needs to be multiplied by 12.
- Cubes with 1 face painted:  $6(n - 2)^2$   
These are in the middle of each face. They are in the shape of a square, two sizes smaller than the face of the original cube. There are 6 faces, so the number needs to be multiplied by 6.
- Cubes with no faces painted:  $(n - 2)^3$   
These are always in the middle. They form a cube shape that is two sizes smaller than the original cube.

# The Painted Cube

**N1 PR2 SS3 SS4**

---

## **Notes:**

This problem not only illustrates linear relationships, but also introduces and reinforces the idea that algebraic relations come from real situations and that can and should be visualized. Students can also graph the different relations.

Assessment idea: Have students initially solve the problem within a group, but individually write up their own solution which explain the process, tells the story of how the problem was solved, and explains the mathematics!

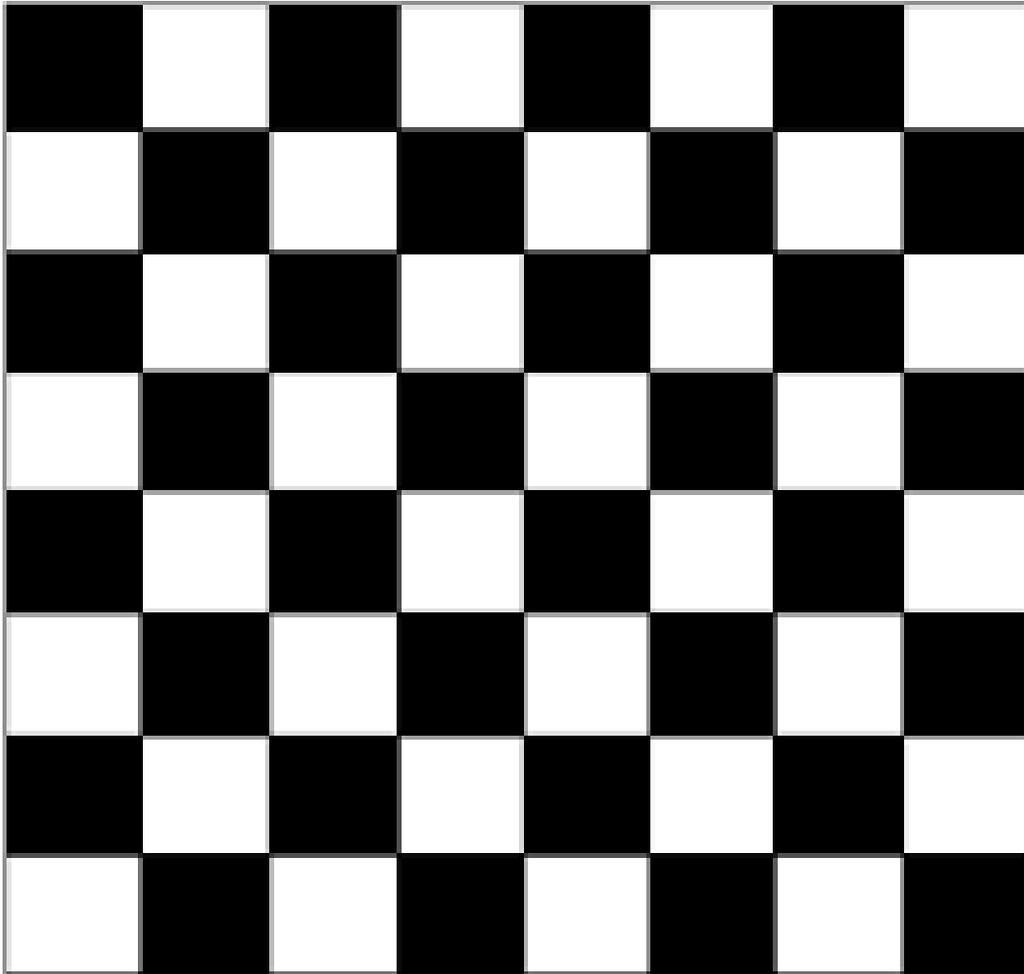
# Checkerboard

**N1**

*\*Credit to Rina Zazkis*

**Problem:**

How many squares are there on a standard 8x8 checkerboard? The answer is not 64!

**Extension:**

If the checkerboard was a different size, could you find a solution?

If the checkerboard was  **$N \times N$**  (any size), could you find an algebraic expression?

What if you had to count the squares that are positioned diagonally (advanced)?

[BACK](#)

# Checkerboard

# N1

## Outcome Objectives:

Number 1-Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]

---

## Material Suggestions:

- Picture of an 8 by 8 checkerboard
  - Scissors to cut out different sized squares
  - Graph Paper and markers
  - Flip Tiles
- 

## Sample Solutions:

$$8^2 + 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 204$$

- Let's say that the checkerboard measures 8cmx8cm.
  - There are 64 1x1cm squares, easily recognized as the red and black squares.
  - There are also 2x2 cm squares, 3x3 cm squares, 4x4 cm squares etc.
  - One way to show this is through the use of graph paper, and students cut out squares measuring 2x2cm, 3x3 cm etc., and moving them along the 8x8 grid.
  - There are 7 – 2x2 cm squares vertically and horizontally, totaling 7<sup>2</sup> 2x2 squares.
  - There are 6 – 3x3 cm squares vertically and horizontally, totaling 6<sup>2</sup> 3x3 squares.
  - Following the same pattern, we reach the equation presented in the solution above.
- 

## Notes:

Students as young as grade three have been successful at solving this problem. Let them struggle and explore.

# Open Lockers

**N1****Problem:**

Imagine that you are in a school that has a row of 100 closed lockers. Suppose a student goes along and opens every locker. Then a second student goes along and shuts every second locker. Now a third student changes the state of every third locker (if the locker is open the student closes it, and if the locker is closed, the student opens it). A fourth student changes the state of every fourth locker. This continues until 100 students have followed this pattern with the lockers.

When finished, which lockers are open? How do you know?

Can you determine the list of open lockers all the way to 1000?

---

**Extension:**

[BACK](#)

# Open Lockers

**N1**

---

**Outcome Objectives:**

Number 1—Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]

---

**Material Suggestions:**

- Flip Tiles or two coloured counters
  - Lockers and sticky notes
  - Markers
- 

**Sample Solution:**

**All square numbered lockers are left open (1,4,9,16,25,...)**

- The first person opens all the lockers, then locker 1 is no longer touched.
- The second person closes every second locker, so all even numbered lockers are closed. Locker 2 is no longer touched.
- The third person changes the state of every third locker, so locker 3 is now closed, and no longer touched.
- The only way a locker can remain open is if it is not touched an even number of times.
- Only numbers with an uneven number of factors are the square numbers.

---

**Notes:**

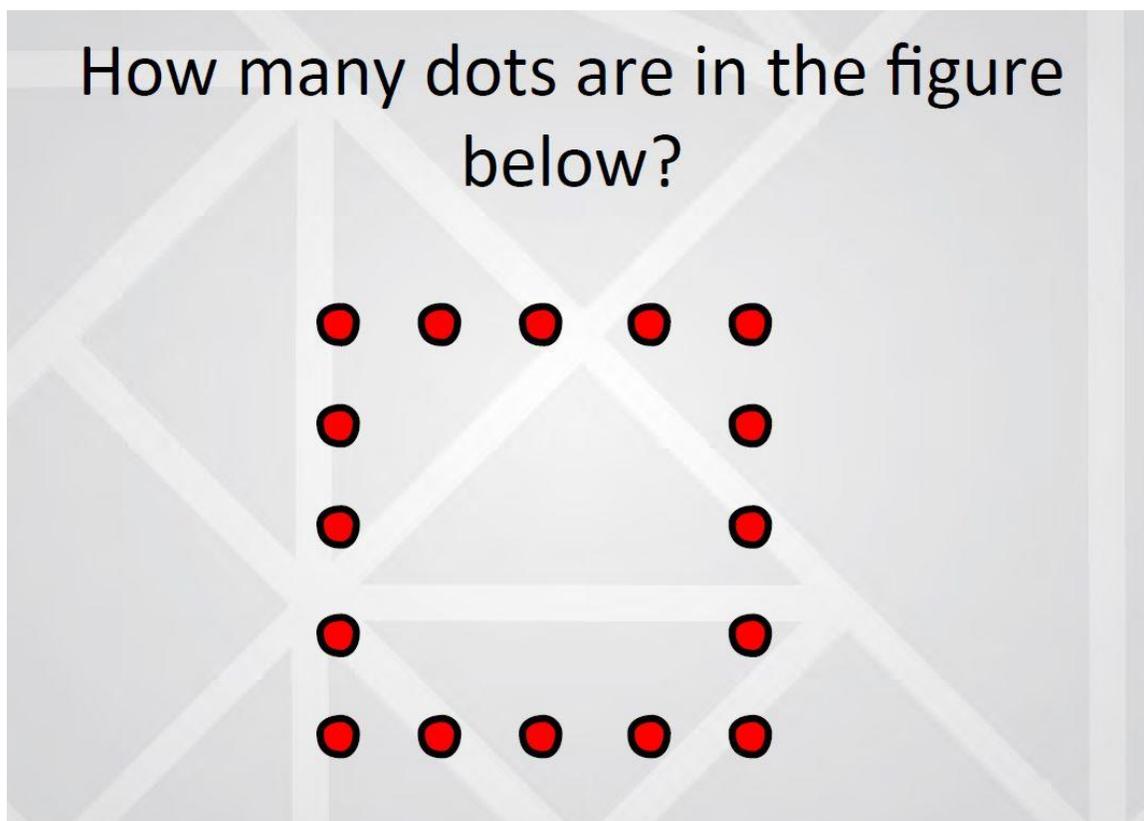
Video - <http://ed.ted.com/lessons/can-you-solve-the-locker-riddle-lisa-winer>

## How Many Dots?

N1 PR1 PR2

*\*Credit to NCTM*

### Problem:



How many dots are in the figure? How do you know? What mathematical sentences can you come up with to help you count the dots?

### Extension:

What if the side lengths were different? Can you predict the number of dots for any side length? Can you find an algebraic expression that represents how you count the dots? What about other algebraic expressions?

[BACK](#)

## How Many Dots?

N1 PR1 PR2

### Outcome Objectives:

Number 1-Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2-3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

$$\bullet ax = b \bullet \frac{x}{a} = b, a \neq 0 \bullet ax + b = c \bullet \frac{x}{a} + b = c, a \neq 0 \bullet a(x + b) = c$$

where  $a$ ,  $b$  and  $c$  are integers. [C, CN, PS, V]

### Material Suggestions:

- counters

### Sample Solutions:

$$4 \times 3 = 12 + 4 = 16$$

How many dots?

$$x^2 - (x-2)^2$$

$$5^2 - 3^2$$

$$25 - 9 = 16$$

Answers will vary.

### Notes:

## Discount Machines

**N3**

*\*Credit to Kordemsky: The Moscow Puzzles (Dover)*

### Problem:

One invention saves 30% on fuel; a second, 45%; and a third, 25%. If you use all three inventions at once can you save 100%? If not, how much?

---

### Extension:

[BACK](#)

## Discount Machines

## N3

### Outcome Objectives:

Number 3-Demonstrate an understanding of percents greater than or equal to 0%, including greater than 100% (CN, PS, R, V).

### Material Suggestions:

### Sample Solutions:

Handwritten calculations on a whiteboard:

$$100$$

$$\hookrightarrow 45\% = 55\%$$

$$\begin{array}{r} \leftarrow 30\% \\ 38.50 \\ - (16.50) \\ \hline \end{array}$$

$$\begin{array}{r} 25\% \rightarrow \\ 28.87 \\ - (9.63) \\ \hline \end{array}$$

$$100 - 28.87$$

Savings of 71.13%

### Notes:

# Percent Benchmarks

# N3

*\*Credit to Wheatley and Abshire. Developing Mathematical Fluency.*

## Percent Benchmarks # 1

1.

100%	50%	25%	10%	5%	$2\frac{1}{2}\%$	1%
1000						

a)  $75\% \times 1000 = \underline{\hspace{2cm}}$       b)  $15\%$  of 1000 is  $\underline{\hspace{2cm}}$

c)  $35\% \times 1000 = \underline{\hspace{2cm}}$       d)  $60\%$  of 1000 is  $\underline{\hspace{2cm}}$

e)  $20\% \times 1000 = \underline{\hspace{2cm}}$       f)  $30\%$  of 1000 is  $\underline{\hspace{2cm}}$

g)  $51\% \times 1000 = \underline{\hspace{2cm}}$       h)  $26\%$  of 1000 is  $\underline{\hspace{2cm}}$

i)  $99\% \times 1000 = \underline{\hspace{2cm}}$       j)  $105\%$  of 1000 is  $\underline{\hspace{2cm}}$

Make up two of your own.

k)  $\underline{\hspace{2cm}} \times 1000 = \underline{\hspace{2cm}}$       l)  $\underline{\hspace{2cm}}$  of 1000 is  $\underline{\hspace{2cm}}$

---

**Extension:**

[BACK](#)

# Percent Benchmarks

# N3

## Sample Solutions:

**Percent Benchmarks # 1**

100%	50%	25%	10%	5%	$2\frac{1}{2}\%$	1%
1000	500	250	100	50	25	10

1. a)  $75\% \times 1000 = \underline{750}$     b) 15% of 1000 is 150

c)  $35\% \times 1000 = \underline{350}$     d) 60% of 1000 is 600

e)  $20\% \times 1000 = \underline{200}$     f) 30% of 1000 is 300

g)  $51\% \times 1000 = \underline{510}$     h) 26% of 1000 is 260

i)  $99\% \times 1000 = \underline{990}$     j) 105% of 1000 is 1050

Make up two of your own.

k) \_\_\_\_\_  $\times$  1000 = \_\_\_\_\_    l) \_\_\_\_\_ of 1000 is \_\_\_\_\_

Notes: Please see Developing Mathematical Fluency for more about fractions, decimals, percents. <http://www.mathematicslearning.org/index.cfm?ref=20200>

## Amazing Offer

**N4 N5**

---

**Problem:**

Your rich friend needs your help with her business for 30 days. She makes you an offer to pay you 1¢ the first day, 2¢ the second day, 4¢ the third day, each day doubling the previous day's pay. Of course, you were insulted and were about to refuse when she said, "Ok, ok, how about \$1,000.00 a day with a \$1,000.00 raise each day." You gladly accept and your friend bursts out laughing. What is so funny?

---

**Extension:**

Is there a point where the first offer is actually better?

Make up another problem that involves exponential growth.

Research a real world situation that deals with exponential growth.

[BACK](#)

## Amazing Offer

N4 N5

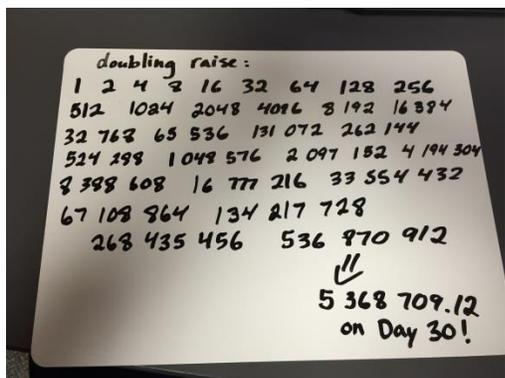
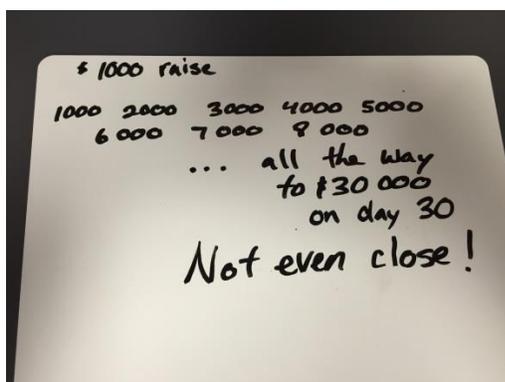
### Outcome Objectives:

Number 4 - Demonstrate an understanding of ratio and rate. [C, CN, V]

Number 5 – Solve problems that involve rates, ratios and proportional reasoning.

[C, CN, PS, R]

### Sample Solutions:



Total salary for the first option is \$10,737,418.23

Total for the second option is \$465,000.00

### Notes:

# Cell Phone Plans

## N4 N5

*\*Credit to Peter Liljedahl*

### Problem:

Your parents have agreed to buy you a cell phone. However, the deal is that you have to pay for the cell phone plan out of your own pocket. There are three plans to choose from:

#### **PAY AS YOU GO**

This truly is “pay as you go”. Phone calls are 25¢ a minute and text messages are 15¢ for each message sent.

#### **BASIC PLAN**

This plan is \$20.00 per month and includes 100 free minutes of “anytime” talk. If you use more than 100 minutes then you will be charged 30¢ per minute for each minute over. Text messages are 15¢ for each message sent.

#### **EASY 4 U PLAN**

This plan is \$50.00 per month and includes 200 weekday minutes and unlimited weekend minutes of talk time. Each additional minute is 35¢. The first 100 text messages sent are free. Anything above that is 25¢ for each message sent.

Show all of the work that leads to your decision.

Write an explanation that will convince your parents that this is the best plan for you.

---

### Extension:

[BACK](#)

## Cell Phone Plans

## N4 N5

---

### Outcome Objectives:

Number 4-Demonstrate an understanding of ratio and rate. [C,CN,V]

Number 5-Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

---

### Material Suggestions:

---

### Sample Solutions:

The solution completely rests upon the assumptions made of air time and text messages, and the best plan will vary depending upon this information.

---

### Notes:

Please see Peter Liljedahl's website for more Numeracy Tasks:

<http://www.peterliljedahl.com/teachers/numeracy-tasks>

# The Jeweler

**N4 N5**

*\*Credit to Malba Tahan, The Man Who Counted, p.23*

---

## Problem:

“This man is a mathematician?” asked old Salim. “Then he has come at just the right moment to help me out of a difficult spot. I have just had a dispute with a jeweler. We argued for a long time, and yet we still have a problem that we cannot resolve.”

“This man,” old Salim said, pointing to the jeweler “came from Syria to sell precious stones in Baghdad. He promised he would pay 20 dinars for his lodgings if he sold all of his jewels for 100 dinars, and 35 dinars if he sold them for 200. After several days of wandering about, he ended up selling all of them for 140 dinars. How much does he owe me according to our agreement?”

---

## Extension:

[BACK](#)

## The Jeweler

## N4 N5

### Outcome Objectives:

Number 4 - Demonstrate an understanding of ratio and rate.

[C, CN, V]

Number 5 – Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

### Material Suggestions:

### Sample Solutions:

Better for Salim  
 $\frac{20}{100} = 0.20$  or 20%  
 $140 \cdot 0.20 = 28 \text{ dinar}$

---

Better for Syrian  
 $\frac{35}{200} = 0.175$  or 17.5%  
 $140 \cdot 0.175 = 24.5 \text{ dinar}$

---

Per 100 dinar interval  
 $100 \cdot 0.20 = 20 \text{ dinar}$   
 $+ 40 \cdot 0.15 = 6 \text{ dinar}$   
 $26 \text{ dinar}$

---

As there is a 2.5% difference between 17.5% + 20% you could look at it as per 20 dinar you pay 0.5% less thus 19%  
 $140 \text{ dinar} \cdot 19\% = 26.6 \text{ dinar}$

---

Add Numerator. Add Divisor use that role  
 $\frac{20+35}{100+200} = \frac{55}{300} = 18.3\%$   $140 \cdot 0.183 = 25.6 \text{ dinar}$

### Notes:

Great problem to discuss assumptions and reasons for different answers.

# 12 Cookies

**N4 N5**

*\*Credit to Geri Lorway*

**Problem:**

Four friends buy 42 cookies for \$12. Each person contributes the following amount of money:

Tom: \$2.00

Jake: \$3.00

Ted: \$4.00

Sam: \$3.00

How many cookies should each person get?

**Extension:**

What if there were more cookies? What if there were more friends?

[BACK](#)

# 12 Cookies

# N4 N5

## Outcome Objectives:

Number 4 – Demonstrate an understanding of ratio and rate.

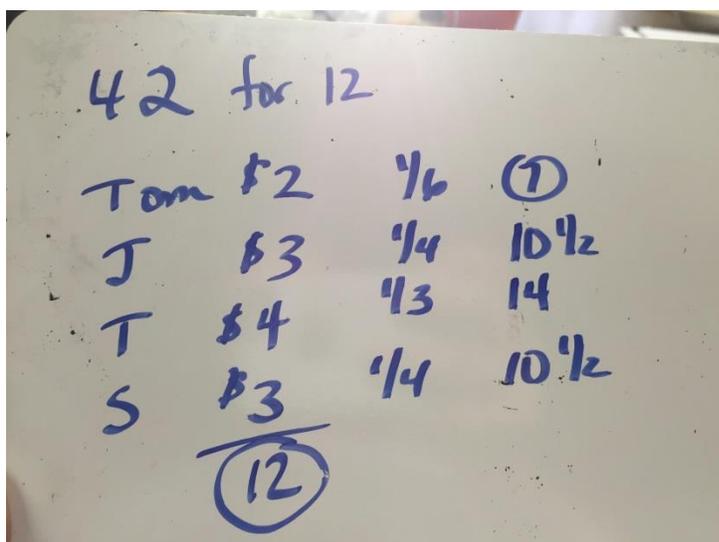
[C, CN, V]

Number 5 – Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

## Material Suggestions:

- Manipulatives

## Sample Solutions:



## Notes:

# Bathtub

**N4 N5 N6**

*\*Credit to Steven Strogatz*

---

**Problem:**

An old fashioned bathtub has two faucets, a hot water tap and a cold water tap.

The hot water tap can fill the tub in half an hour.

The cold water tap has a blockage and does not run properly, so it takes one hour to fill the tub.

If the two taps run concurrently, how long will it take to fill the tub?

---

**Extension:**

It takes three minutes to fill a tub to the top and five minutes to drain the full tub.

If the faucet and drain are both open, how long will it take to fill the tub?

[BACK](#)

# Bathtub

## N4 N5 N6

---

### Outcome Objectives:

Number 4 – Demonstrate an understanding of ratio and rate.

[C, CN, V]

Number 5 – Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Number 6 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

---

### Material Suggestions:

---

### Sample Solution:

If the hot water tap takes 30 minutes to fill the tub, it fills  $\frac{1}{3}$  of the tub in 10 minutes. The cold water takes 60 minutes to fill the tub, so in ten minutes it can fill  $\frac{1}{6}$  of the tub. If both taps run for 10 minutes, the tub would be half full ( $\frac{1}{6} + \frac{1}{3} = \frac{3}{6}$ ). We would need to double the time to fill the whole tub. It will take a total of 20 minutes.

---

### Notes:

# Retirement

**N4 N5 N6**

*\*Credit to Rina Zazkis*

---

**Problem:**

An assortment of single people and married couples are living in a retirement complex.

$\frac{2}{5}$  of the men are married to  $\frac{3}{4}$  of the women

How many men might there be? How many women? How many different answers can you find?

---

**Extension:**

Try with different fractions.

[BACK](#)

# Retirement

# N4 N5 N6

## Outcome Objectives:

Number 4 – Demonstrate an understanding of ratio and rate.

[C, CN, V]

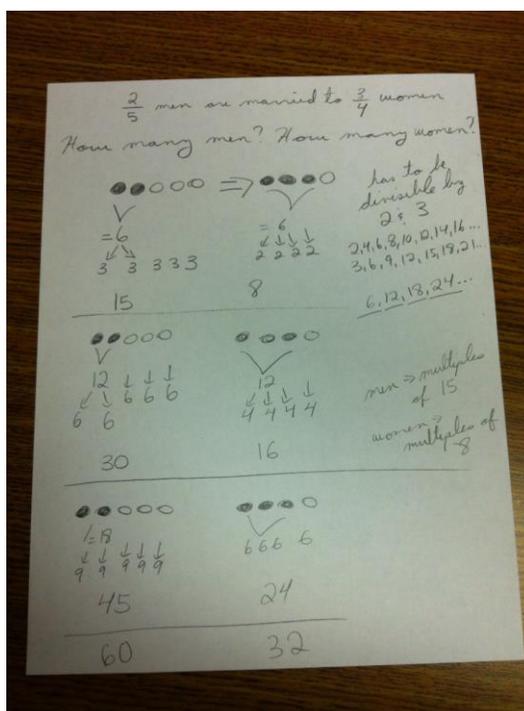
Number 5 – Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Number 6 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

## Material Suggestions:

- Manipulatives

## Sample Solutions:



## Notes:

# Bungee Barbie

N4 N5 PR1 PR2

*\*Credit to Peter Liljedahl*

## Problem:



How many linked elastics do you need for Barbie to bungee jump from a determined distance? Her hair can touch the ground, but not her head. You can only test your prediction twice.

You will need data, a table of values, and a graph as part of your solution before you are allowed to test.

---

## Extension:

[BACK](#)



# Grannie Stitchwork

**N4 N5**

*\*Credit to Mathematical Cavalcade by Brian Bolt, Cambridge University Press, 1992. ISBN 978-0-521-42617-6*

**Problem:**

Grannie Stitchwork made 20 teddy bears each month to sell in the local toyshop. She received a monthly delivery of 6 square metres of fur material, 5 kg of stuffing, 4 metres of coloured ribbon for bow ties and 40 special eye buttons. She made 20 identical teddies.

The teddies sold well but customers were asking for teddies twice the size, so Grannie was asked to make 20 large teddies for the following month. Always obliging, Grannie Stitchwork immediately doubled the order of materials for the next month.

How many large teddy bears will she be able to make after doubling her material order?

---

**Extension:**

[BACK](#)

# Grannie Stitchwork

## N4 N5

---

### Outcome Objectives:

Number 4-Demonstrate an understanding of ratio and rate. [C,CN,V]

Number 5- Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

---

### Material Suggestions:

---

### Sample Solutions:

She made 5 large teddies.

- The ribbon would take double 4 m or 8 m for the larger bears. That works out.
  - The 6 m<sup>2</sup> of fur would require 4x as much, or 24 m<sup>2</sup>, because doubling the linear dimensions increases the surface area by a factor of 4. (If the original is 2x3 then doubling those dimensions is 4x6=24 or 4x the amount.)
  - The stuffing is a volume measurement, which increases by a factor of 8 when the linear dimensions are doubled which would require 5x8=40kg of stuffing. (If the original is 2x2x2, then doubling becomes 4x4x4 – from 8 to 64, or 8x as large).
  - Her new order is 12 m<sup>2</sup> of fur, 10kg of stuffing and 8m of ribbon. She actually needs 24m<sup>2</sup> of fur, 40 kg of stuffing and 16 m of ribbon. The stuffing is the most limiting, allowing her only ¼ of the order to be filled, or 5 teddies
- 

### Notes:

# Pennies

**N5 N6**

---

**Problem:**

There are some pennies on a table. One fourth of the pennies are heads up. If two pennies are turned over then one third of the pennies are heads up.

How many pennies are on the table? How do you know?

---

**Extension:**

Create your own problem. What if your numerators could not be one?

[BACK](#)

# Pennies

**N5 N6**

---

**Outcome Objectives:**

Number 5-Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

---

**Material Suggestions:**

- pennies
  - other manipulatives
- 

**Sample Solution:**

24 pennies because  $\frac{1}{4}$  of 24 is 6 and  $\frac{1}{3}$  of 24 is 8.

---

**Notes:**

# Traffic Lights

## SP2 N5

---

*\*Credit to NCTM*

### Problem:

A complete cycle of a traffic lights takes 60 seconds. During each cycle the light is yellow for 5 seconds and red for 30 seconds. At a randomly chosen time, what is the probability that the light will be green?

---

### Extension:

[BACK](#)

# Traffic Lights

## SP2 N5

### Outcome Objectives:

Statistics and Probability (Chance and Uncertainty) 2-Solve problems involving the probability of independent events.

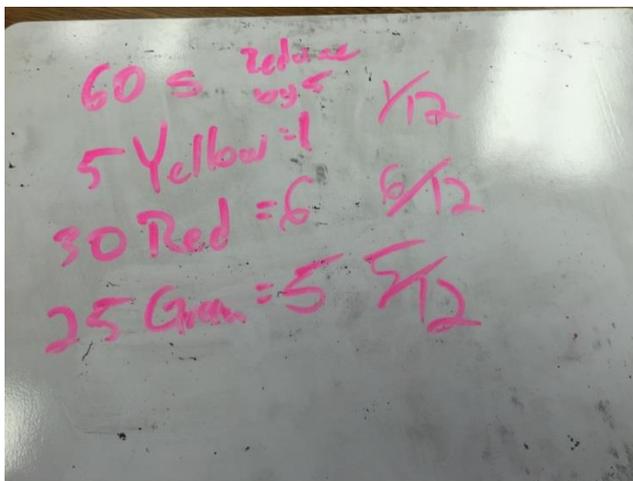
[C, CN, PS, T] [ICT: P2–3.4]

Number 5-Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

### Material Suggestions:

### Sample Solutions:

25/60 or 42% probability that the light will be green.



### Notes:

# Bears and Apples

**N6**

*\* Credit to The Story of the Three Bears (adapted from Teaching Mathematics as Storytelling by Rina Zazkis and Peter Liljedahl).*

## Problem:

“Once upon a time there were three bears named Minnie, Mickey, and Molly. One sunny day the bears went for a walk in the forest. They played games, picked berries and were enjoying themselves so much that they lost all sense of time and were very surprised when it suddenly got dark. It was so dark that they could not find their way home. So they wandered around until they became very tired and very hungry. They sat under a tree to get some rest, and they all fell asleep. At that time a kind fairy was passing by. She saw the three bears and thinking that they looked hungry she left them a basket of apples. In the middle of the night Minnie, the oldest bear woke up. Seeing the basket of tasty red apples she thought, “What a wonderful treat, these apples look so good and I’m so hungry. I want to eat them all.” But then she remembered that she was not alone and that her siblings were likely hungry as well. She also remembered what mama bear had taught her about sharing. So Minnie only ate one-third of the apples and immediately fell back to sleep.

Another hour passed by and Mickey woke up. He saw the basket of tasty red apples and thought, “What a wonderful treat, these apples look so good and I’m so hungry. I want to eat them all.” But then he remembered that he was not alone, and his siblings were likely hungry as well. He also remembered what mama bear had taught him about sharing. So Mickey only ate one-third of the apples and immediately fell back to sleep. Another hour passed by and Molly woke up. She was so hungry that she ate her apples and went back to sleep.

Slowly, the forest awoke to the sunny morning. The birds were singing and their lovely songs woke the three sleeping bears. They saw a basket under the tree.

**What was in the basket?”**

---

## Extension:

If there were 8 apples left in the basket when the bears awoke in the morning, how many apples did the Fairy leave them?

What if there was a different amount of apples left in the basket?

[BACK](#)

# Bears and Apples

## N6

### Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

### Material Suggestions:

- Basket
- Manipulatives or apples

### Sample Solutions:

#### 27x apples

- If each bear eats  $\frac{1}{3}$  of the apples, and there is a whole number of apples in the basket to begin with, then bear #3 eats  $\frac{1}{3}$  of  $\frac{1}{3}$  of  $\frac{1}{3}$  of the apples, or  $\frac{1}{27}$ <sup>th</sup> of the apples.
- Any multiple of 27 can be divided in 3 and then in 3 and then in 3 again.

if each bear eats  $\frac{1}{3}$ , then  
 the last bear has eaten  
 $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$  of the apples.  
 so there are 27 apples to begin  
 with. - or a multiple of 27.

start	#1 eats $\frac{1}{3}$	# left	#2 eats $\frac{1}{3}$	# left	#3 eats $\frac{1}{3}$	# left
27	9	18	6	12	4	8
54	18	36	12	24	8	12
81	27	54	18	36	12	24

Alternately, if we assume that bear#3 eats all the apples left in the basket, then there are only  $9x$  apples to begin with, as bear 1 and bear 2 each  $\frac{1}{3}$ , meaning that the number must divide by 3 and then 3 again. If this is the case, then 9 apples to start goes to 6 apples then to 4 apples that the last bear consumes.

### Notes:

## Birds on a Lake

**N6**

---

**Problem:**

A flock of geese are on a lake.  $\frac{1}{5}$  of the geese fly away.  $\frac{1}{8}$  of the remaining birds are startled and fly away too. Finally, three times the number that first left now fly away. Twenty-eight geese are left on the lake. How many were there to begin with?

---

**Extension:**

[BACK](#)

## Birds on a Lake

**N6**

---

### Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

---

### Material Suggestions:

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### Sample Solutions:

280 birds were on the lake to begin with.

- $1/5$  flew away first.
  - $1/8(4/5)$  flew away next.
  - $3(1/5)$  flew away last.
  - $1/5 + 1/8(1/5) + 3(1/5) =$  total fraction that flew away. So  $1/5 + 1/40 + 3/5 = 8/40 + 1/40 + 24/40 = 33/40$
  - if  $33/40$  flew away, then  $7/40$  stayed.
  - $7/40 = 28/n$ , n being the total number on the lake.
  - $N=280$ .
- 

### Notes:

## Cookie Jar

**N6**

*\*Credit to Geri Lorway*

---

### **Problem:**

There was a jar of cookies on the table. First Ken came along. He was hungry because he missed lunch, so he ate half the cookies in the jar. Mary came in the kitchen next. She didn't have dessert at lunch so she ate one third of the cookies left in the jar. Karen followed her to the cookie jar and she took three fourths of the remaining cookies. Susan ran into the kitchen, grabbed one cookie from the jar and ran out again. That left one cookie in the jar. How many were in the jar to begin with?

---

### **Extension:**

Could there be another number of cookies?

[BACK](#)

## Cookie Jar

**N6**

### Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

---

### Material Suggestions:

- Snap Cubes
  - Pattern Blocks
  - Graph Paper
- 

### Sample Solution:

**There were 24 cookies in the jar to begin with.**

- Working backwards, there were 2 cookies in the jar before Susan took one.
  - 2 cookies is  $\frac{1}{4}$  of the batch that Karen ate (she ate  $\frac{3}{4}$  of the cookies in the jar at that point). So that means there were 8 cookies in the jar, Karen ate 6 of them, and left 2.
  - The 8 cookies left represent  $\frac{2}{3}$  of the cookies because Mary ate  $\frac{1}{3}$  of the batch, leaving  $\frac{2}{3}$  there for Karen.  $8 = \frac{2}{3}$  of what? 12. That means Mary ate 4 cookies, leaving 8 in the jar.
  - Now, 12 cookies is  $\frac{1}{2}$  the batch left after Ken, so that means there were 24 cookies in the jar to start with.
- 

### Notes:

# Fractions/Decimals/Percents

# N3 N6

*\*Credit to Wheatley and Abshire. Developing Mathematical Fluency.*

## Problem:

### Fraction-Decimal-Percent Two Ways # 1

1. Write 0.5 as a percent.
2. Write  $\frac{3}{4}$  as a decimal.
3. Write 0.25 as a fraction in simplest terms.
4. Change 10% to a fraction in simplest terms.
5. Write  $\frac{1}{5}$  as percent.
6. Change 37% to a decimal.

7.  $\textcircled{x}$

	$2\frac{1}{2}$	7.5
50%		
	10	

8.  $\textcircled{x}$

	1.25	
25%		$\frac{6}{3}$
		10

9.  $\textcircled{x}$

$\frac{1}{2}$		
200%	1.5	
		9

10.  $\textcircled{x}$

12	.5	
400%	$4\frac{1}{2}$	

## Extension:

[BACK](#)

# Fractions/Decimals/Percents

## N3 N6

### Outcome Objectives:

Number 3-Demonstrate an understanding of percents greater than or equal to 0%, including greater than 100% (CN, PS, R, V).

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

### Material Suggestions:

### Sample Solutions:

**Fraction-Decimal-Percent Two Ways # 1**

1. Write 0.5 as a percent. 50%
2. Write  $\frac{3}{4}$  as a decimal. 0.75
3. Write 0.25 as a fraction in simplest terms.  $\frac{1}{4}$
4. Change 10% to a fraction in simplest terms.  $\frac{1}{10}$
5. Write  $\frac{1}{5}$  as percent. 20%
6. Change 37% to a decimal. 0.37

7. 

3	$2\frac{1}{2}$	7.5
50%	4	2
1.5	10	15

8. 

4	1.25	5
25%	8	$\frac{6}{3}$
1	10	10

9. 

$\frac{1}{2}$	6	3
200%	1.5	3
1	9	9

10. 

12	.5	6
$\frac{1}{3}$	9	3
400%	$4\frac{1}{2}$	18

**Notes:** See Developing Mathematical Fluency for more about fractions, decimals, percents. <http://www.mathematicslearning.org/index.cfm?ref=20200>

## Sharing Bread

**N6**

*\*Credit to Malba Tahan, The Man Who Counted, p.15*

### **Problem:**

“Three days later, we were approaching the ruins of a small village called Sippar, when we found sprawled on the ground a poor traveler, his clothes in rags and he apparently badly hurt. His condition was pitiful. We went to the aid of the unfortunate man, and he later told us the story of his misfortune. His name was Salem Nasair, and he was one of the richest merchants in Baghdad. On the way back from Basra, a few days before, bound for el-Hilleh, his large caravan had been attacked and looted by a band of Persian desert nomads, and almost everyone had perished at their hands. He, the head, managed to escape miraculously by hiding in the sand. When he had finished his tale of woe, he asked us in a trembling voice, “Do you by some chance have anything to eat? I am dying of hunger. I have three loaves of bread I answered. I have five said the man who counted. Very well answered the sheik. I beg you to share those loaves with me. Let me make an equitable arrangement. I promise to pay for the bread with eight pieces of gold, when I get to Baghdad. So we did...”

The three of them equally shared the eight loaves of bread. How did the rich merchant fairly pay the two strangers with the 8 pieces of gold?

---

### **Extension:**

[BACK](#)

# Sharing Bread

## N6

---

### Outcome Objectives:

Number 6 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

---

### Material Suggestions:

- Snap Cubes
- 

### Sample Solutions:

**1 gold coin to the man with 3 loaves and 7 gold coins to the man with 5 loaves.**

- 3 loaves and 5 loaves divided onto 3 parts yields 24 parts of loaves of bread.
  - Each person eats 8 parts.
  - The man with 3 loaves has 1 part to give to the shiek (  $3 \text{ loaves} \times 3 \text{ parts} = 9 \text{ parts}$ , of which he eats 8, leaving 1 for the shiek)
  - The man with 5 loaves has  $5 \times 3 = 15$ ,  $-8 = 7$  parts for the shiek.
  - The shiek owes  $\frac{1}{8}$  to the man with 3 loaves and  $\frac{7}{8}$  gold to the man with 5 loaves.
- 

### Notes:

## Giving Out Bonuses

## N6

*\*Credit to Peter Liljedahl*

### Problem:

You are the manager of the Text-n-Talk Cell Phone Company that employs a number of independent sales people to sell their phones **seven days a week**. These sales people work as much or as little as they want. As a sales manager you don't care how much they work, but you do care how much they sell. To motivate them to sell more, you give out bonuses based on how productive they have been.

There are two bonus plans:

- the top producing individual receives **\$500**.
- the top producing team **shares \$500 in a fair manner**.

However, there are also two problems:

- different people have different ways of reporting their productivity.
- the individual sales teams don't have the same number of people on them.

Based on the information provided in the table below, **who should get the bonuses this month, and how much do you think they should get? Justify your answers in writing.**

Sales Person	Team	Sales Reported for the Month of April (30 days)
<u>Tysen</u>	A	300 cell phones sold this month
Peter	B	An average of 56 cell phones sold every 5 days
Lewis	A	An average of $10\frac{1}{3}$ cell phones sold each day
<u>Ainsley</u>	A	598 cell phones sold in the last 60 days
Avery	A	An average of $98\frac{3}{4}$ cell phones sold every 10 days
Jennifer	B	An average of $11\frac{4}{15}$ cell phones sold each day
Steven	B	An average of 55 cell phones each week
Gabrielle	C	4113 cell phone sold in the last year
Diana	C	An average of 10.05 cell phones each day
Matthew	D	An average of 10.87 cell phones each day
<u>Alexa</u>	D	An average of $9\frac{1}{6}$ cell phones each day
Jasmine	C	267 cell phones this month

### Extension:

[BACK](#)

## Giving Out Bonuses

---

**N6****Outcome Objectives:**

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

---

**Material Suggestions:**

---

**Sample Solution:**

Solutions will vary.

---

**Notes:**

See Peter Liljedahl's website for more Numeracy Tasks:

<http://www.peterliljedahl.com/teachers/numeracy-tasks>

## Sharing Sub Sandwiches

**N6**

*\*Credit to Cathy Fosnot*

---

### **Problem:**

A fifth-grade class traveled on a field trip in four separate cars. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows:

- the first group had 4 people and shared 3 subs equally.
- the second group had 5 people and shared 4 subs equally.
- the third group had 8 people and shared 7 subs equally.
- the last group had 5 people and shared 4 subs equally.

When they returned from the field trip, the children began to argue that the distribution of sandwiches had not been fair, that some children got more to eat than the others. Were they right?

---

### **Extension:**

[BACK](#)

# Sharing Sub Sandwiches

## N6

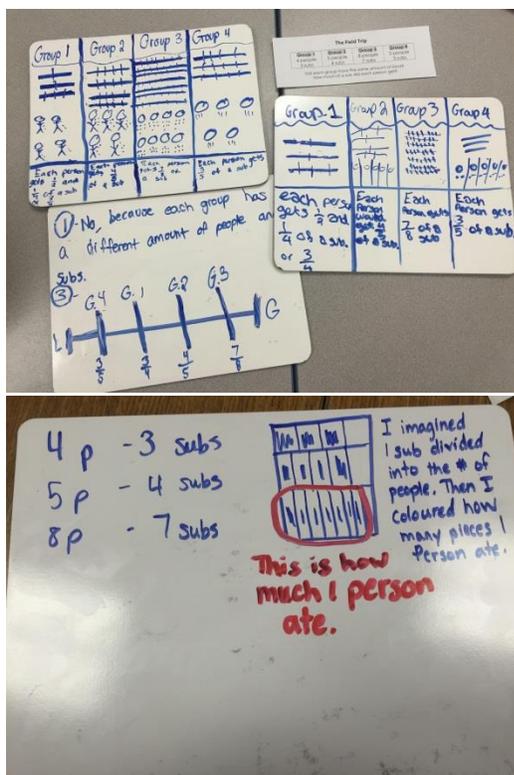
### Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

### Material Suggestions:

- Unifix cubes
- Other manipulatives

### Sample Solutions:



### Notes:

This can be solved in a variety of ways. Students can use fractions, decimals, percents.

## Integers – Teaching Progression 1 of 9

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**N7****Lesson One: Discuss Integers, what they are and how they are used.**

Whole-group brainstorm for ideas followed by some research on the ideas. (Elevation, altitude, airplanes, scuba divers, above or below sea-level, temperatures, bank accounts and overdraft, plus and minus scores in hockey, T-Minus 10, 9, 8...blast off!, golf scores, etc.).

**Exit Card: Explain integers and two places you may use them in everyday life.**

[BACK](#)

# Integers – Teaching Progression 2 of 9

# N7

## Lesson Two (optional): Review Adding and Subtracting Integers

*\*Credit to Wheatley and Abshire, Developing Mathematical Fluency*

### Integer Two Ways # 1

1. 
$$\begin{array}{|c|c|} \hline \oplus & \\ \hline 2 & -4 \\ \hline -8 & 5 \\ \hline \end{array}$$

2. 
$$\begin{array}{|c|c|} \hline \oplus & \\ \hline 4 & -2 \\ \hline -1 & -9 \\ \hline \end{array}$$

3. 
$$\begin{array}{|c|c|} \hline \oplus & \\ \hline -4 & -2 \\ \hline 0 & -1 \\ \hline \end{array}$$

4. 
$$\begin{array}{|c|c|} \hline \oplus & \\ \hline -7 & \\ \hline 2 & -8 \\ \hline 5 & \\ \hline \end{array}$$

5. 
$$\begin{array}{|c|c|} \hline \oplus & \\ \hline -6 & -8 \\ \hline 1 & 4 \\ \hline \end{array}$$

6. 
$$\begin{array}{|c|c|} \hline \oplus & \\ \hline -2 & 3 \\ \hline -3 & -2 \\ \hline \end{array}$$

**Exit Card: Explain how to add and subtract integers. Give multiple examples and explain why your strategies work.**

[BACK](#)

## Integers – Teaching Progression 3 of 9

**N7**

**Lesson Three: Representing Multiplication of Integers with Manipulatives** (integer tiles, bingo chips and pictures (number line) with one positive integer and one negative integer.

$$(+3) \times (-4) = (-4) \times (+3) = (-3) \times (+4) = (+4) \times (-3)$$

$$3 \text{ groups of } (-4) = 4 \text{ groups of } (-3) = (-12)$$

**Exit Card: Explain how to multiply a positive integer and a positive integer.  
Explain how to multiply a positive integer and a negative integer.**

[BACK](#)

## Integers – Teaching Progression 4 of 9

**N7**

### Lesson Four: Exploring Mathematical Statements in Groups

The following mathematical statements are true. Can you figure out others that also must be true? Can you figure out some rules about multiplying and dividing integers? Can you test these rules? Use technology.

$$(-4) \times (-5) = (+20)$$

$$\text{Therefore } (+20) \div (-5) = (-4)$$

$$(-2) \times (+3) = (-6)$$

$$\text{Therefore } (-6) \div (+3) = (-2)$$

$$(+6) \times (-6) = (-36)$$

$$\text{Therefore } (-36) \div (-6) = (+6)$$

$$(+7) \times (+2) = (+14)$$

$$\text{Therefore } (+14) \div (+2) = (+7)$$

**Exit Card: What do you know for sure about multiplying and dividing positive and negative integers?**

[BACK](#)

## Integers – Teaching Progression 5 of 9

**N7**

### Lesson Five: Following the Patterns and Making Conclusions

$4 \times 3 =$
$3 \times 3 =$
$2 \times 3 =$
$1 \times 3 =$
$0 \times 3 =$
$(-1) \times 3 = \underline{\quad}$
$(-2) \times 3 = \underline{\quad}$
$(-3) \times 3 = \underline{\quad}$
$(-4) \times 3 = \underline{\quad}$

$4 \times (-3) =$
$3 \times (-3) =$
$2 \times (-3) =$
$1 \times (-3) =$
$0 \times (-3) =$
$(-1) \times (-3) = \underline{\quad}$
$(-2) \times (-3) = \underline{\quad}$
$(-3) \times (-3) = \underline{\quad}$
$(-4) \times (-3) = \underline{\quad}$

**Exit Card: What conclusions can you make about multiplying positive and negative integers? What about dividing positive and negative integers?**

[BACK](#)

## Integers – Teaching Progression 6 of 9

## N7

### Lesson Six: A Story to Explain

*\*Credit to Rinal Zazkis and Peter Liljedahl, Teaching Mathematics as Storytelling by Sense Publishers, 2009).*

#### Represent the following situations with a math equation.

“We consider a chemical reaction in which the temperature is rising by 2 degrees every hour. The current temperature is 0. What will the temperature be in 5 hours”?

“Consider a chemical reaction in which the temperature is increasing by 2 degrees every hour. The current temperature is 0. What will the temperature be in 5 hours”?

“Consider a chemical reaction in which the temperature is increasing by 2 degrees every hour. The current temperature is 0. What was the temperature 5 hours ago”?

“Consider a chemical reaction in which the temperature is decreasing by two degrees every hour. The current temperature is 0. What was the temperature 5 hours ago”?

**Exit Card: Can we find or invent other situations that could represent the same math equations?**

[BACK](#)

# Integers – Teaching Progression 7 of 9

# N7

## Lesson Seven: Multiplying and Dividing Integers

*\*Credit to Wheatley and Abshire, Developing Mathematical Fluency*

### Integer Two Ways # 7

1.  $\otimes$

-4	-3	
	-2	
20		

2.  $\otimes$

		2
-5	-5	
-10		

3.  $\otimes$

	-4	
	-3	-6
		48

4.  $\otimes$

-7	1	
	4	
-21		

5.  $\otimes$

-5		-50
	4	
		-200

6.  $\otimes$

2	-3	
		-7
	-21	

**Exit Card: Explain how to add and subtract integers. Give multiple examples and explain why your strategies work.**

[BACK](#)

## Integers – Teaching Progression 8 of 9

---

**N7**

### Lesson Eight: What about division?

Can we make sense of division with integers? Can we find and invent examples of situations where we divide with positive and negative numbers?

**Exit Card: Find or invent situations that represent different divisions with integers (positive and negative).**

[BACK](#)

## Integers – Teaching Progression 9 of 9

## N7

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**Lesson Nine: Showcase your learning using technology to prove the following statement:**

I can “demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically”.

---

### Notes:

Obviously, these aren't really “problems” but a way to teach integers with a problem solving approach! Students need to explore, experiment, try, and verify together in groups to figure out integers. Silly rhymes or rules may help them succeed on a test, but without understanding or long term learning. Let them use bingo chips, number lines, thermometers, etc. to help make sense of operations with integers.

The *Show Me App for the iPad* is a great way to showcase learning and understanding, explaining thinking, etc. Another option is to have the students make a video as they explore/explain/teach how to multiply and divide integers.

Visit <http://gdaymath.com/lessons/powerarea/1-4-an-aside-on-negative-numbers-piles-and-holes/> for James Tanton's aside on negative numbers.

[BACK](#)

# Algebra Sort 1

## N6 N7

*\*Credit to Geri Lorway, Marj Farris, NRLC*

### Problem:

Sort each of the following 12 statements into the categories: Always True, Sometimes True, and Never True. Be sure to discuss your thoughts with your group. Be prepared to display your agreed upon reasoning in a poster format. Include examples or reasons for each statement.

Previous Knowledge: Cut out and sort these statements into the categories given.		
Always True	Sometimes True	Never True
$2n + 3 = 3 + 2n$	$n + 5$ is less than 20	$2t - 3 = 3 - 2t$
$5q = 5$	$2x = 2x$	$4p$ is greater than $9 + p$
$2 \cdot 3 + s = 6 + s$	$k + 12 = g + 12$	$d + 3 = d \div 3$
$2x = 4$	$q + 2 = q + 16$	$n + 5 = 11$

## Algebra Sort 2

## N6 N7

Sort each of the following 12 statements into the categories: Always True, Sometimes True, and Never True. Be sure to discuss your thoughts with your group. Be prepared to display your agreed upon reasoning in a poster format. Include examples or reasons for each statement.

Grade 8: Cut out and sort these statements into the categories given.		
Always True	Sometimes True	Never True
$6x = 2(3x)$	$n + 5$ is less than 20	$x^2 = 9$
$2n + 3 = 3 + 2n$	$n + 5 = 11$	$k + 12 = g + 12$
$2(3 + s) = 6 + 2s$	$4p$ is greater than $9 + p$	$2(d + 3) = 2d + 3$
$2t - 3 = 3 - 2t$	$q + 2 = q + 16$	$y * y = 2y$

## Algebra Sort 3

## N6 N7

Sort each of the following 12 statements into the categories: Always True, Sometimes True, and Never True. Be sure to discuss your thoughts with your group. Be prepared to display your agreed upon reasoning in a poster format. Include examples or reasons for each statement.

Extension: Cut out and sort these statements into the categories given.		
Always True	Sometimes True	Never True
$9x^2 = (3x)^2$	$n + 5$ is less than 20	$2t - 3 = 3 - 2t$
$2n + 3 = 3 + 2n$	$n + 5 = 11$	$q + 2 = q + 16$
$2(3 + s) = 6 + 2s$	$4p$ is greater than $9 + p$	$2(d + 3) = 2d + 3$
$k + 12 = g + 12$	$x^2$ is greater than 4	$x^2 = 5x$

# Algebra Sort

## N6 N7

### Outcome Objectives:

Number 6-Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Number 7-Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.  
[C, CN, PS, R, V]

### Material Suggestions:

- Algebra Sort number cards pre-cut

### Sample Solutions:

Previous knowledge

Always	Sometimes	Never
$2x = 2x$	$n + 5$ is less than 20	$2t - 3 = 3 - 2t$
$2n + 3 = 3 + 2n$	$n + 5 = 11$	$q + 2 = q + 16$
$2 \cdot 3 + s = 6 + s$	$4p$ is greater than $9 + p$	$d + 3 = d + 3$
	$2x = 4$	
	$5q = 5$	
	$k + 12 = g + 12$	

Grade 8

Always	Sometimes	Never
$6x = 2(3x)$	$n + 5$ is less than 20	$2t - 3 = 3 - 2t$
$2n + 3 = 3 + 2n$	$n + 5 = 11$	$q + 2 = q + 16$
$2(3 + s) = 6 + 2s$	$4p$ is greater than $9 + p$	$2(d + 3) = 2d + 3$
	$x^2 = 9$	
	$y \cdot y = 2y$	
	$k + 12 = g + 12$	

Extension:

Always	Sometimes	Never
$9x^2 = (3x)^2$	$n + 5$ is less than 20	$2t - 3 = 3 - 2t$
$2n + 3 = 3 + 2n$	$n + 5 = 11$	$q + 2 = q + 16$
$2(3 + s) = 6 + 2s$	$4p$ is greater than $9 + p$	$2(d + 3) = 2d + 3$
	$x^2$ is greater than 4	
	$x^2 = 5x$	

### Notes:

Have the statements pre-cut and mixed up. The students could make a poster or other representation of their finished sort and explain why they sorted the way they did.

# Moving Squares

**PR1 PR2**

*\*Credit to David Pimm*

---

## Problem:

You have 16 people in a group filling a 4 x 4 grid (4 rows of 4). If you remove the person in the top right corner, how can you move the person in the bottom left corner to that empty space? You can only move vertically or horizontally, not diagonally. How many moves will this take? Is this the minimum number of moves?

---

## Extension:

What about in different sizes of grids?

How do we calculate the minimum number of moves for any size grid?

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# Moving Squares

PR1 PR2

## Outcome Objectives:

Patterns and Relations 1 - Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V]

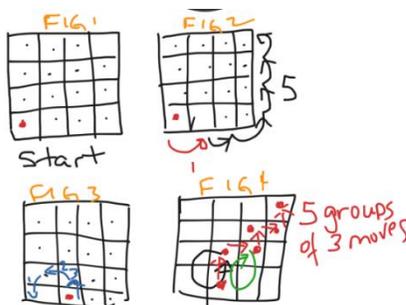
Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:  $ax = b$   $\frac{x}{a} = b$ ,  $a \neq 0$   $ax + b = c$   $\frac{x}{a} + b = c$ ,  $a \neq 0$   $a(x + b) = c$  where  $a$ ,  $b$  and  $c$  are integers.

[C, CN, PS, V]

## Material Suggestions:

- People in a grid (made of tape on the floor if desired)
- Cubes or square counters
- Graph paper

## Sample Solutions:



It takes 6+ (3x5)

**moves =21 moves**

- First must move the empty space adjacent to the person moving, in this picture, the Cheerio. On a 4x4 grid, this takes 5 moves, plus 1 more move of the Cheerio
- Then the dance of 3 begins – it takes 5 dances of 3 steps to move it to the correct corner.

	Move the blank and move the person once	#of sets of 3 dances	total
2x2	2	1	5
3x3	4	3	13
4x4	6	5	21
5x5	8	7	29

**8x-11 comes from somewhere in the movement of the squares, where?**

**(This is complex and may not be reached except by the most motivated and mathematically ready).**

## Notes:

Take your time to act it out as a class and spend time with the manipulatives and problem solving. It is worth it!

## Growing Pattern

PR1 PR2

*\*Credit to NCTM*

### Problem:



What changes as this pattern grows?

Use a table, graph, equation, and story to describe the change.

---

### Extension:

[BACK](#)

# Growing Pattern

# PR1 PR2

## Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

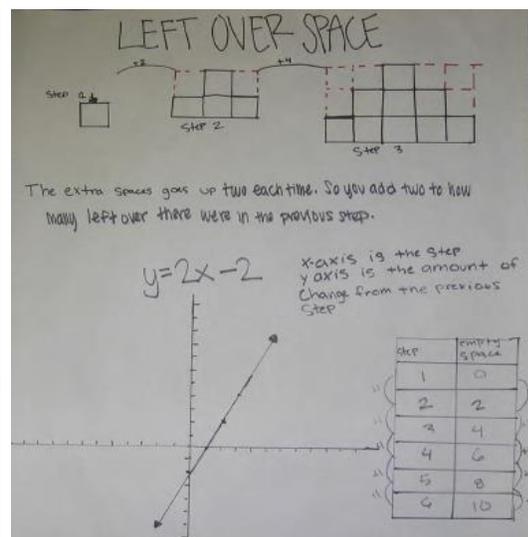
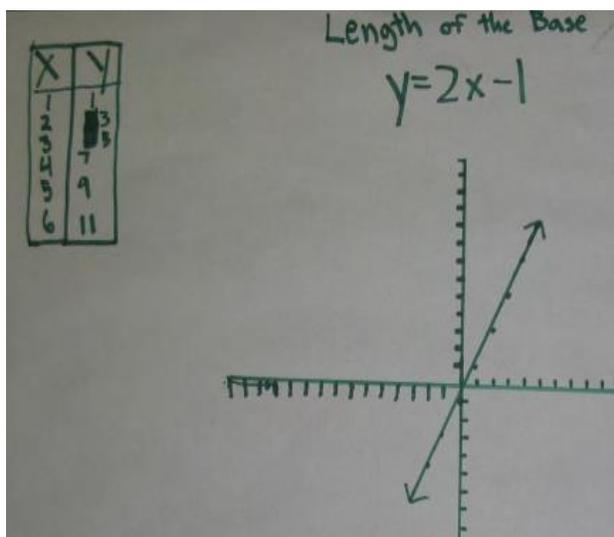
Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

•  $ax = b$  •  $\frac{x}{a} = b$ ,  $a \neq 0$  •  $ax + b = c$  •  $\frac{x}{a} + b = c$ ,  $a \neq 0$  •  $a(x + b) = c$  where  $a$ ,  $b$  and  $c$  are integers. [C, CN, PS, V]

## Material Suggestions:

- toothpicks
- graph paper

## Sample Solutions:



## Notes:

Have students come up with ideas to investigate (ie. perimeter, height, width, size of enclosing rectangle, # of toothpicks, # of interior toothpicks, # of intersections, # of corners, # of squares, # of rectangles, # of parallel lines, # of diagonals, left over space, etc.

# Mountain Range

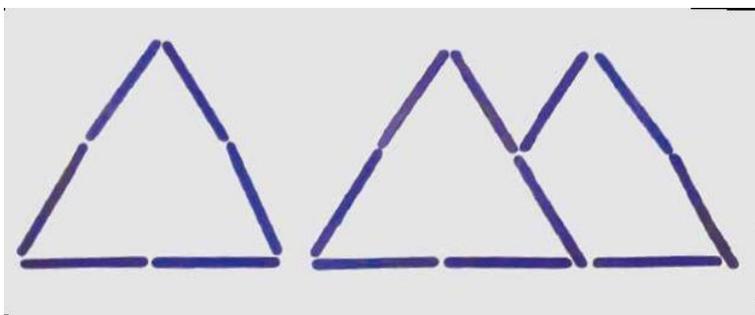
**PR1 PR2***\*Credit to NCTM*

## Problem:

Using popsicle sticks, create a mountain range of equilateral triangles. First predict and then determine how many sticks will be needed to build a mountain range with 5 peaks, 8 peaks and 10 peaks. How many sticks would be needed to build a mountain range with 100 peaks?

Use isometric dot paper to create a diagram that represents your mountain range models. Be sure to record your response in a systematic manner and demonstrate on your model how you solved the problem.

Can you determine how many sticks would be needed to build a model of a mountain range with any number of peaks?



---

## Extension:

[BACK](#)

# Mountain Range

## PR1 PR2

### Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

•  $ax = b \cdot \frac{x}{a} = b, a \neq 0$  •  $ax + b = c \cdot \frac{x}{a} + b = c, a \neq 0$  •  $a(x + b) = c$  where  $a, b$  and  $c$  are integers. [C, CN, PS, V]

### Material Suggestions:

- toothpicks or popsicle sticks
- triangular dot paper

### Sample Solutions:

Figure	# popsicle sticks
1	6
2	10
3	14
4	18
5	22

- 4 more popsicle sticks are added each time
- If the first figure is the constant (6 sticks), then the chart looks like this:

Figure (n)	constant	Additional sticks
1	6	0
2	6	4
3	6	8
4	6	12
n	6	4(n-1)

### Notes:

There are a few different expressions, so answers may vary.

# Picture Frames

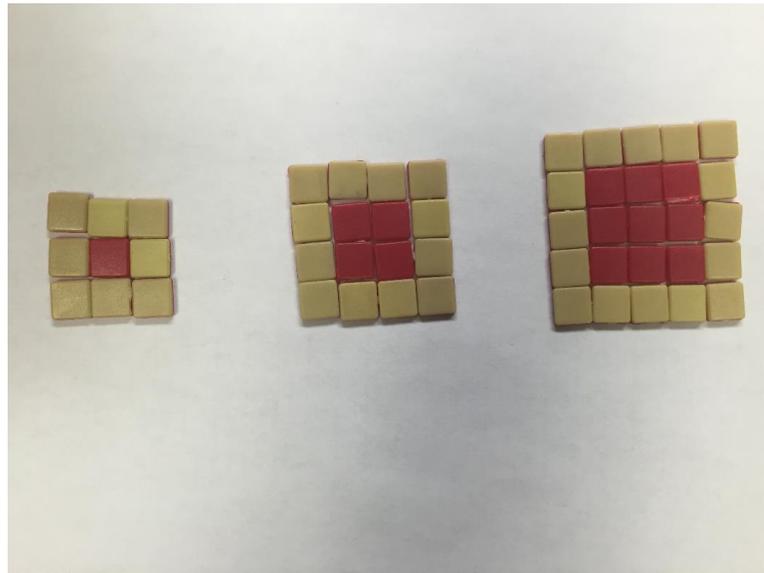
**PR1 PR2**

*\*Credit to Mathematics Assessment Sampler, Grades 6-8, NCTM*

## Problem:

Build a sequence of squares with a different colour border.

What different patterns are there? How can you represent these different patterns using a table, a graph and algebra?



---

## Extension:

[BACK](#)

## Picture Frames

## PR1 PR2

### Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

•  $ax = b$  •  $\frac{x}{a} = b$ ,  $a \neq 0$  •  $ax + b = c$  •  $\frac{x}{a} + b = c$ ,  $a \neq 0$  •  $a(x + b) = c$  where  $a$ ,  $b$  and  $c$  are integers. [C, CN, PS, V]

### Material Suggestions:

- square tiles
- graph paper
- markers

### Sample Solutions:

This exploration is wide open. Students may look at the inside tiles, the border tiles or the total tiles.

For example, the number of inside tiles are square numbers. This can be shown in a chart or a sequence of numbers.

The total tiles is  $(n+2)^2$  when  $n$  represents the figure number.

The number of outside tiles is  $(n+2)^2 - n^2$ . Total tiles less the inside tiles.

### Notes:

## Pool Border

PR1 PR2

*\*Credit to David Pimm*

### Problem:

Find the number of one-by-one tiles required to surround a 4 by 3 pool. Find a rule to predict the number of tiles required to surround a rectangular pool of any size. See if you can express the rule as a visual representation and as an expression.



---

### Extension:

What if the pool was not rectangular?

[BACK](#)

## Pool Border

## PR1 PR2

### Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

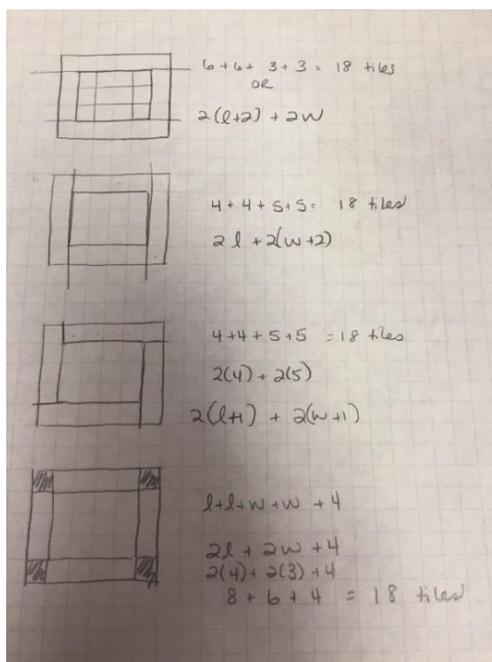
Patterns and Relations (Variables and Equations) 2-Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

•  $ax = b$  •  $\frac{x}{a} = b$ ,  $a \neq 0$  •  $ax + b = c$  •  $\frac{x}{a} + b = c$ ,  $a \neq 0$  •  $a(x + b) = c$  where  $a$ ,  $b$  and  $c$  are integers. [C, CN, PS, V]

### Material Suggestions:

- tiles
- graph paper

### Sample Solutions:



### Notes:

See sample solution first for the connection between the different expressions and their visual representations.

# Staircase

**PR1 PR2**

*\*Credit to Vol. 108, No. 9 • May 2015 | MATHEMATICS TEACHER 663*

## Problem:

A group of students are building staircases out of wooden cubes. The 1-step staircase consists of one cube, and the 2-step staircase consists of three cubes stacked (see fig. 1).

How many cubes will be needed to build a 3-step staircase? A 6-step staircase? A 50-step staircase? An  $n$ -step staircase?



---

## Extension:

What about these?



[BACK](#)

# Staircase

## PR1 PR2

### Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

•  $ax = b$  •  $\frac{x}{a} = b$ ,  $a \neq 0$  •  $ax + b = c$  •  $\frac{x}{a} + b = c$ ,  $a \neq 0$  •  $a(x + b) = c$  where  $a$ ,  $b$  and  $c$  are integers. [C, CN, PS, V]

### Material Suggestions:

- unifix cubes or blocks
- graph paper

### Sample Solutions:

The sample solutions show several ways to approach the staircase problem:

- Top Left:** A staircase diagram with 6 steps. The number of blocks in each step is labeled: 1, 2, 3, 4, 5, 6.
- Top Middle:** Three smaller staircase diagrams with 3, 6, and 10 blocks respectively.
- Top Right:** A table showing the relationship between the number of steps and the total number of cubes.
 

# of steps	# of cubes
1	1
2	3
3	6
4	10
5	15
6	21
7	28
8	36
9	45
10	55
50	2,500
- Top Far Right:** A list of equations:  $7 = 28$ ,  $8 = 36$ ,  $9 = 45$ ,  $10 = 55$ , and  $50 = 2500$ .
- Bottom Left:** A staircase diagram where each step is represented by a circle, and arrows point to the circles with numbers 1 through 6.
- Bottom Middle:** A diagram of a staircase with 50 steps. Text next to it says: "50 steps = 50 blocks", "50 x 50 = 2500", and "Blocks" circled.
- Bottom Right:** A graph on a grid showing a staircase pattern.

### Notes:

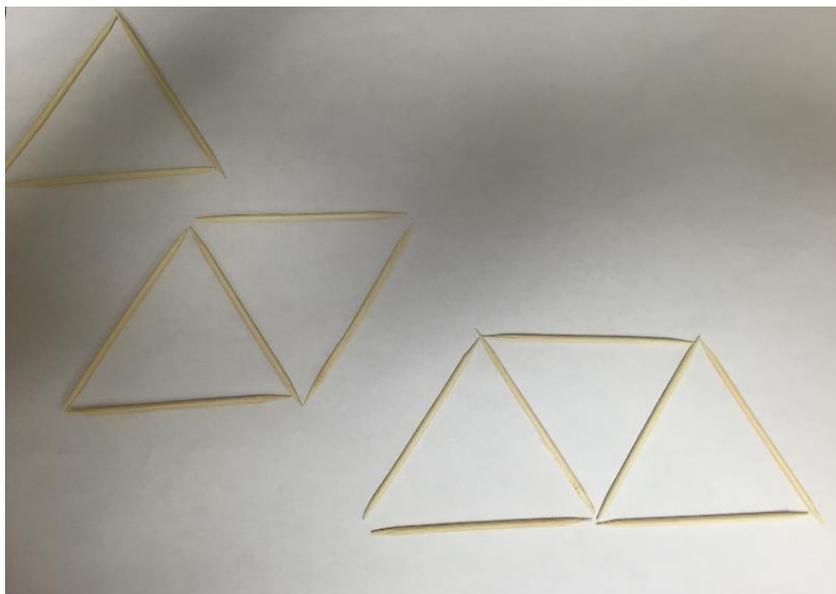
# Toothpicks

## PR1 PR2

*\*Credit to Mathematics Assessment Sampler, Grades 6-8, NCTM*

### Problem:

While waiting for my food at a restaurant I began to play with toothpicks. This is what I came up with:



Then I started wondering about my pattern...

How many toothpicks are in each figure?

How is my pattern growing?

How would I know how many toothpicks I would need for the next structure?

If I followed the pattern, which one would require 27 toothpicks to build?

Can I find an expression that represents the rule for the number of toothpicks in Figure  $n$ ?

---

### Extension:

Make and investigate another toothpick pattern.

[BACK](#)

# Toothpicks

## PR1 PR2

### Outcome Objectives:

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

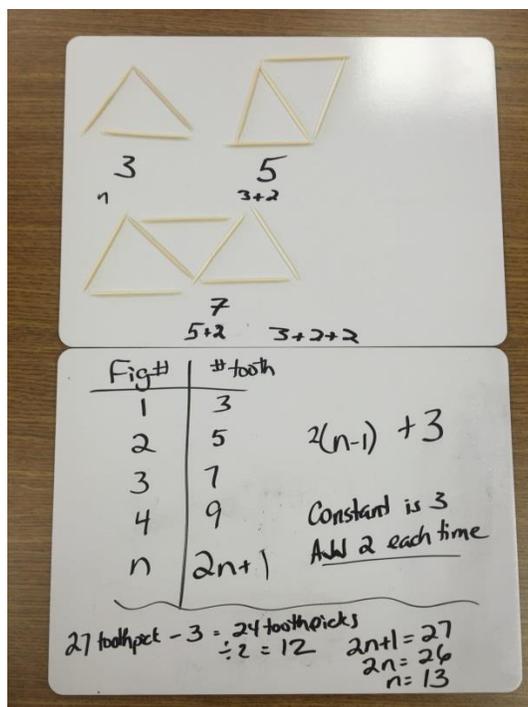
Patterns and Relations (Variables and Equations) 2-Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

•  $ax = b$  •  $\frac{x}{a} = b, a \neq 0$  •  $ax + b = c$  •  $\frac{x}{a} + b = c, a \neq 0$  •  $a(x + b) = c$  where  $a, b$  and  $c$  are integers. [C, CN, PS, V]

### Material Suggestions:

- toothpicks
- triangular dot paper

### Sample Solutions:



### Notes:

# Skyscraper Windows

## PR1 PR2 SS2

---

*\*Credit to Driscoll*

### Problem:

A building is 12 stories high and is covered entirely by windows on all four sides. Each floor has 38 windows on it. Once a year, all the windows are washed. The cost for washing each window depends upon the window's floor:

Floor	Cost per window
1	\$2.00
2	\$2.50
3	\$3.00
...	...

For example, each window on Floor 2 costs \$2.50 to wash. This pricing scale continues for the windows on each floor.

1. How much will it cost to wash all the windows of this building?
2. If the building is 30 stories tall, how much will it cost to wash all the windows?
3. If the building is  $n$  stories tall, how much will it cost to wash all the windows?

---

### Extension:

[BACK](#)

# Skyscraper Windows

## PR1 PR2 SS2

### Outcome Objectives:

Shape and Space (Measurement) 2-Draw and construct nets for 3-D objects. [C, CN, PS, V]

Patterns and Relations (Patterns) 1-Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V] [ICT: P2–3.3]

Patterns and Relations (Variables and Equations) 2- Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:

•  $ax = b$  •  $\frac{x}{a} = b$ ,  $a \neq 0$  •  $ax + b = c$  •  $\frac{x}{a} + b = c$ ,  $a \neq 0$  •  $a(x + b) = c$   
 where  $a$ ,  $b$  and  $c$  are integers. [C, CN, PS, V]

### Material Suggestions:

### Sample Solutions:

1. It would cost \$2394.00 to wash the windows of this building.

Floor	Cost per Floor	Cost of 38 windows
1	\$2.50	\$95.00
2	\$3.00	\$114.00
3	\$3.50	\$133.00
4	\$4.00	\$152.00
5	\$4.50	\$171.00
6	\$5.00	\$190.00
7	\$5.50	\$209.00
8	\$6.00	\$228.00
9	\$6.50	\$247.00
10	\$7.00	\$266.00
11	\$7.50	\$285.00
12	\$8.00	\$304.00
<b>Total</b>		<b>\$2,394.00</b>

A pairing strategy can also be used:

6 groups of \$10.50 x 38 windows (cost of 12<sup>th</sup> floor plus 1<sup>st</sup> floor is \$10.50, 2<sup>nd</sup> floor plus 11<sup>th</sup> floor is \$10.50...)

## Skyscraper Windows

## PR1 PR2 SS2

### Sample Solutions Continued:

2. For 30 windows there would be 15 pairs of  $\$19.50 \times 38 = \$11\ 115.00$

3. n windows would cost ... $\$2.00 + \$0.50n$  where n is the floor.

There is a basic constant of \$2.00, plus an increase of 50¢ per floor, expressed as  $\$0.50n$ .

Floor	Cost per Floor	Cost of 38 Windows	Constant	.50n	Cost per Floor
1	\$2.50	\$95.00	2	\$0.50	\$2.50
2	\$3.00	\$114.00	2	\$1.00	\$3.00
3	\$3.50	\$133.00	2	\$1.50	\$3.50
4	\$4.00	\$152.00	2	\$2.00	\$4.00
5	\$4.50	\$171.00	2	\$2.50	\$4.50
6	\$5.00	\$190.00	2	\$3.00	\$5.00
7	\$5.50	\$209.00	2	\$3.50	\$5.50
8	\$6.00	\$228.00	2	\$4.00	\$6.00
9	\$6.50	\$247.00	2	\$4.50	\$6.50
10	\$7.00	\$266.00	2	\$5.00	\$7.00
11	\$7.50	\$285.00	2	\$5.50	\$7.50
12	\$8.00	\$304.00	2	\$6.00	\$8.00
<b>Total</b>		\$2,394.00			

---

**Notes:**

## 51- Foot Ladder

**SS1**

*\*Credit to Mathalicious*

---

### **Problem:**

Is a 51-foot ladder tall enough to get over a 50-foot wall?  
Prove your solution mathematically.

What is a safe ladder angle? What are the safety recommendations?  
How far away do you place the base of the ladder from the wall?

Ladder Trucks can rescue people from different floors of a building. Some ladder trucks have 100-foot ladders. What floor would you want to stay on to ensure that you could be rescued if there was a fire?

How high are ceilings? How tall is the ladder truck?

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### **Extension:**

How tall would your ladder need to be to get over a 50-foot wall?

[BACK](#)

## 51- Foot Ladder

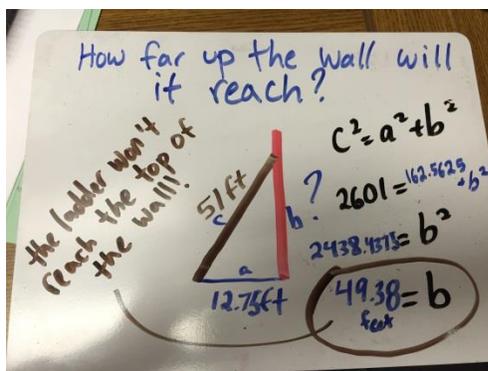
## SS1

### Outcome Objectives:

Shape and Space (Measurement) 1-Develop and apply the Pythagorean Theorem to solve problems. [CN, PS, R, T, V]  
[ICT: P2–3.4]

### Material Suggestions:

### Sample Solutions:



Solutions will vary, especially for the ladder truck part of the problem which depends largely on assumptions (height of ladder truck, ceiling heights of building, etc.). Lots of discussion needed during and after.



### Notes:

Visit <http://www.mathalicious.com/> for more real world lessons!

# Pringles

**SS1 SS3**

*\*Credit to Peter Liljedahl*

## Problem:

Create a paper cone that fits exactly inside a snack-size Pringles can. You will be able to exactly fit the circumference of your cone in the bottom of the can and the point will touch the lid exactly.



## Extension:

What if it was a regular sized Pringles can?

Can you make a Pringle Ringle?



Visit <https://youtu.be/0v9n34CckSo> to see one being built.

[BACK](#)

# Pringles

## SS1 SS3

### Outcome Objectives:

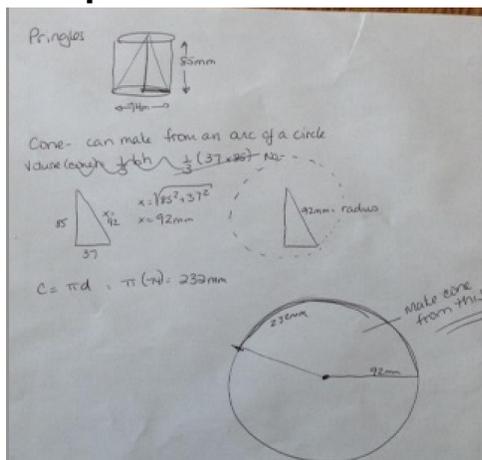
Shape and Space 1- Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]

Shape and Space 3- Determine the surface area of: • right rectangular prisms • right triangular prisms • right cylinders to solve problems. [C, CN, PS, R, V]

### Material Suggestions:

- Small Pringles Can Snack Size
- Printer paper
- Compass
- Ruler, scissors, tape

### Sample Solutions:



- The key is to recognize first that a circle of some sort is needed to form the cone.
- Next is to realize that the hypotenuse of the triangle formed by the height of the pringles' container and the half the diameter is actually the radius of the circle needed.
- Finally, the whole circle is not required – only the length of arc equal to the circumference of the pringles container – this can be measured with a string.
- Cut out that piece of the circle, and voila! – tape the sides of the cone and insert into container.

### Notes:

## How Much Popcorn?

**SS2 SS3 SS4**

*\*Credit to Figure This!*

### Problem:

Which holds more popcorn? A cylinder made of a rectangular piece of paper rolled widthwise or a cylinder made of the same size of paper rolled lengthwise? Explain why.

---

### Extension:

What if the containers weren't cylinders?

[BACK](#)

# How Much Popcorn

**SS2 SS3 SS4**

---

**Outcome Objectives:**

Shape and Space 2 – Draw and construct nets for 3-D objects. [PS, R, T, V]

Shape and Space 3 – Determine the surface area of: right rectangular prisms, right triangular prisms, right cylinders to solve problems. [C, CN, PS, V]

Shape and Space 4 – Develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms, and right cylinders. [C, CN, PS, R, V]

---

**Material Suggestions:**

- Popcorn
  - Rectangular paper
- 

**Sample Solutions:**

The students will discover that even though the surface area of the two cylinders is the same, their volumes are different. The shorter cylinder will hold more popcorn because the surface area of the base is larger.

---

**Notes:**

# Pentominoes

**SS5 SS6**

*\*Credit to Peter Liljedahl and Wikipedia*

## Problem:

A **pentomino** is a plane geometric figure formed by joining five equal squares edge to edge. **Using snap cubes, how many different 3-D “pentominoes” can you make?** (The pentomino obtained by [reflecting](#) or [rotating](#) a pentomino does not count as a different pentomino).



## Extension:

Can you tile a rectangular box with each pentomino? Can you tile a rectangular box with a mix of pentominoes? Can you use all of the pentominoes once to fill a rectangular box?

[BACK](#)

# Pentominoes

## SS5 SS6

### Outcome Objectives:

Shape and Space 5 – Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms.

[C, CN, R, T, V]

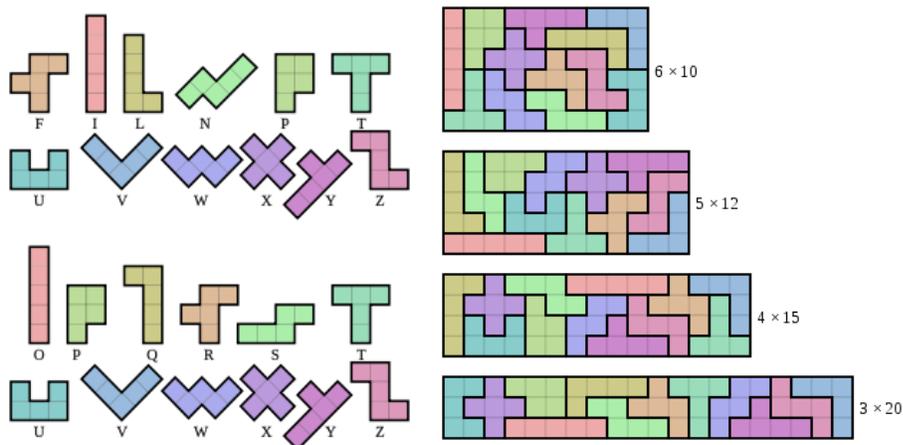
Shape and Space 6 – Demonstrate an understanding of the congruence of polygons.

[CN, R, V]

### Material Suggestions:

- Snap cubes
- Masking Tape to make a rectangle

### Sample Solutions:



### Notes:

More information, challenges, sample solutions:

<http://www.mathematische-basteleien.de/pentominos.htm>

# Eyewitness

## SP2

*\*Credit to James Tanton*

---

### Problem:

In a certain town, 90% of the cars are purple, 10% are blue. A crime is committed. An eye-witness says the thief drove off in a blue car. Testing shows that the witness can correctly identify colour 80% of the time. What is the probability that the car really was blue?

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### Extension:

[BACK](#)

# Eyewitness

# SP 2

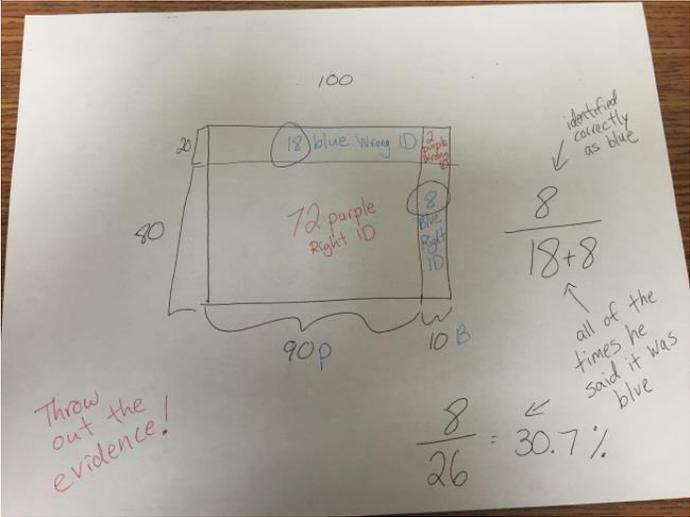
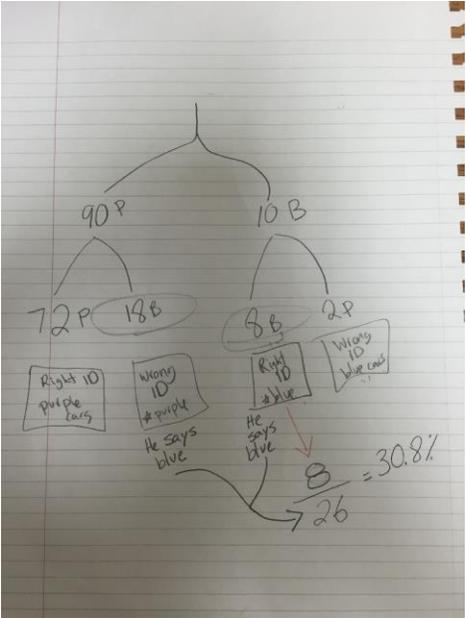
### Outcome Objectives:

Statistics and Probability 2 – Solve problems involving the probability of independent events. [C, CN, PS, R]

### Material Suggestions:

- Manipulatives

### Sample Solutions:



### Notes:

# Hat and Rabbit

**SP2**

*\*Credit to Ian Stewart*

---

**Problem:**

A magician has a hat with a rabbit in it. The rabbit is either black or white. She then places a white rabbit into the hat to join the other rabbit. What is the probability that a white rabbit is left in the hat after one has been pulled out?

---

**Extension:** What if there was a different number of rabbits?

[BACK](#)

# Hat and Rabbit

## SP2

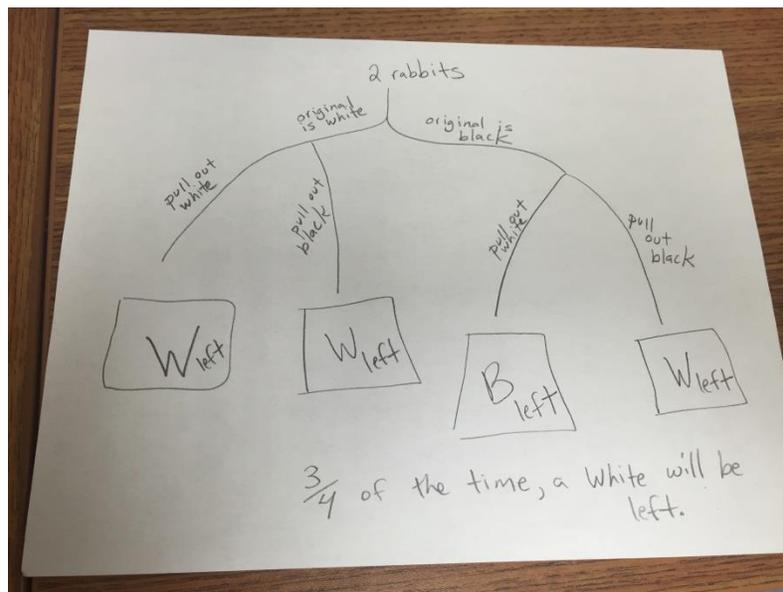
### Outcome Objectives:

Statistics and Probability 2 – Solve problems involving the probability of independent events. [C, CN, PS, R]

### Material Suggestions:

- A top hat
- Rabbits

### Sample Solution:



### Notes:

# The Monte Hall Problem

**SP2**

*\*Credit to American Mathematical Monthly, January 1992*

---

**Problem:**

A TV host shows you three numbered doors (all three equally likely), one hiding a car and the other two hiding goats. You get to pick a door, winning whatever is behind it. Regardless of the door you choose, the host, who knows where the car is, then opens one of the other two doors to reveal a goat, and invites you to switch your choice if you so wish. Does switching increase your chances of winning the car?

Should you stick or switch? Why?

---

**Extension:**

[BACK](#)

# The Monte Hall Problem

## SP2

---

### Outcome Objectives:

Statistics and Probability 2 – Solve problems involving the probability of independent events. [C, CN, PS, R]

---

### Material Suggestions:

- Manipulatives
  - Plastic Cups to use as doors
  - Online simulation: <http://www.grand-illusions.com/simulator/montysim.htm>
- 

### Sample Solutions:

The probability of winning the car is  $1/3$  if you stick and  $2/3$  if you switch, so you should always switch.

Assume that you always start by picking Door #1, and the host then always shows you some other door which does *not* contain the car, and you then always switch to the remaining door.

If the car is behind Door #1, then after you pick Door #1, the host will open another door (either #2 or #3), and you will then switch to the remaining door (either #3 or #2), thus LOSING.

If the car is behind Door #2, then after you pick Door #1, the host will be forced to open Door #3, and you will then switch to Door #2, thus WINNING.

If the car is behind Door #3, then after you pick Door #1, the host will be forced to open Door #2, and you will then switch to Door #3, thus WINNING.

Hence, in 2 of the 3 (equally-likely) possibilities, you will win. Ergo, the probability of winning by switching is  $2/3$ .

(\*Sample solution from [Jeffrey S. Rosenthal](#), 2006)

---

### Notes:

# The Restaurant

# N5 PR2

Problem:

**The Restaurant Problem**

Three waiters delivered three different meals. The cost of each meal is shown. What is the cost of each item?

The diagram shows three waiters holding trays of food. The top waiter holds a burger and a glass of orange juice, with a price tag of £12. The bottom-left waiter holds two burgers and a pineapple, with a price tag of £34. The bottom-right waiter holds a pineapple and a glass of orange juice, with a price tag of £19.

**Extension:** Invent your own “Restaurant Problem”.

[BACK](#)

## The Restaurant

## N5 PR2

### Outcome Objectives:

Number 5-Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Patterns and Relations (Variables and Equations) 2-Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:  $ax = b$ ,  $\frac{x}{a} = b$ ,  $a \neq 0$ ,  $ax + b = c$ ,  $\frac{x}{a} + b = c$ ,  $a \neq 0$ ,  $a(x + b) = c$  where  $a$ ,  $b$  and  $c$  are integers.

[C, CN, PS, V]

### Material Suggestions:

### Sample Solution:

2 hamburgers and one pineapple plus  
1 hamburger and 1 OS  
 $\text{€ } 34 + \text{€ } 12 = \text{€ } 46$   
3 Burgers + 1 Pineapple + 1 OS = 46 Euros  
Subtract 19 Euros (1 pineapple + 1 OS)  
= 27 Euros for 3 hamburgers  
 $\therefore$  9 Euros for 1 hamburger

$H + OS = 12$      $9 + OS = 12$      $OS = 3$  Euros  
 $2H + P = 34$      $2(9) + P = 34$      $P = 16$  Euros  
 $P + OS = 19$      $16 + 3 = 19$  ✓     $H = 9$  Euros

### Notes:

# Cap and Umbrella

## N5 PR2

*\*Credit to Mark Driscoll, Fostering Algebraic Thinking, A Guide for Teachers Grades 6-10*

**Problem:**

**What is the price of one umbrella? One cap?**

The image displays two rows of items and their corresponding prices. The top row consists of one red cap, two rainbow umbrellas, and a yellow price tag labeled "\$80.00". The bottom row consists of one rainbow umbrella, two red caps, and a yellow price tag labeled "\$76.00".

---

**Extension:** Invent your own "Cap and Umbrella" Problem.

[BACK](#)

# Cap and Umbrella

## N5 PR2

### Outcome Objectives:

Number 5-Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Patterns and Relations (Variables and Equations) 2-Model and solve problems concretely, pictorially and symbolically, using linear equations of the form:  $ax = b$ ,  $\frac{x}{a} = b$ ,  $a \neq 0$ ,  $ax + b = c$ ,  $\frac{x}{a} + b = c$ ,  $a \neq 0$ ,  $a(x + b) = c$  where  $a$ ,  $b$  and  $c$  are integers. [C, CN, PS, V]

### Material Suggestions:

- Manipulatives

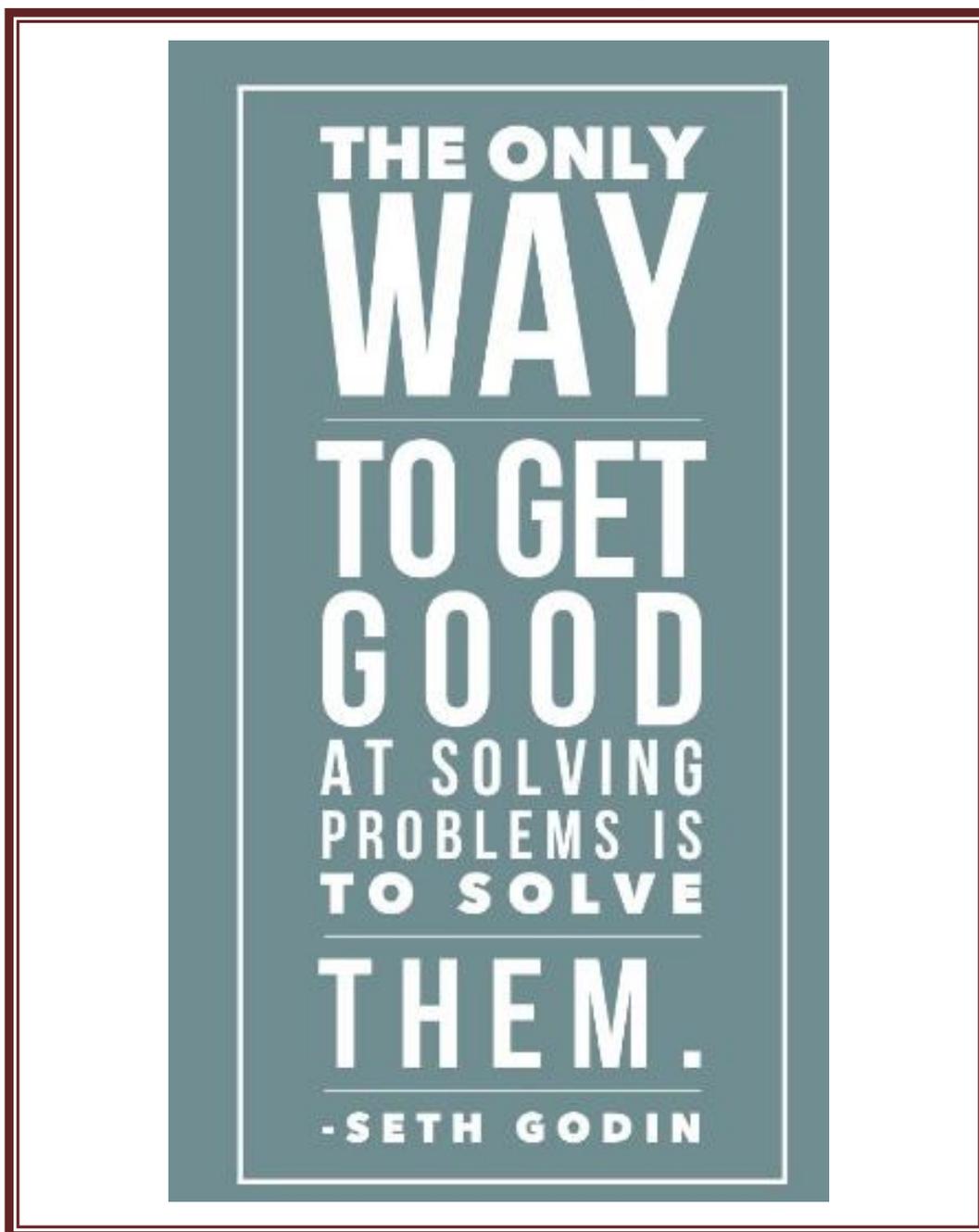
### Sample Solution:

Ways to solve:

- cover up a cap and an umbrella from each row and the difference in price between cap and umbrella is \$4.00
  - see two rows as pattern and add third row below with 3 caps at \$72.00. divide to find 1 cap.
  - same pattern but add row above with 3 umbrellas at \$84.00. divide to find 1 umbrella.
  - create system of equations,  $2x + 1y = 80$  and  $x + 2y = 76$
  - might add them together to get  $3x + 3y = 156$ . divide by 3 to get  $x + y = 52$
- we don't need to go to deep algebraic equations to solve something that only requires reasoning  
 answer: cap = \$24 and umbrella = \$28

### Notes:

## New Problems



**For Your Thinking Classroom**

## New Problems for Your Thinking Classroom

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## Six Js and Dinner Rolls

---

### Problem:

Jack, Jenny, Jeanie, Julie, Jerry, and Jeremy are all attending a dinner party. Jack doesn't like Jenny, Jeanie doesn't like Jeremy, Julie loves Jerry, and Jenny always steals Julie's dinner rolls. If they are sitting at a round table, who would you seat to Jeremy's left?

---

### Extension:

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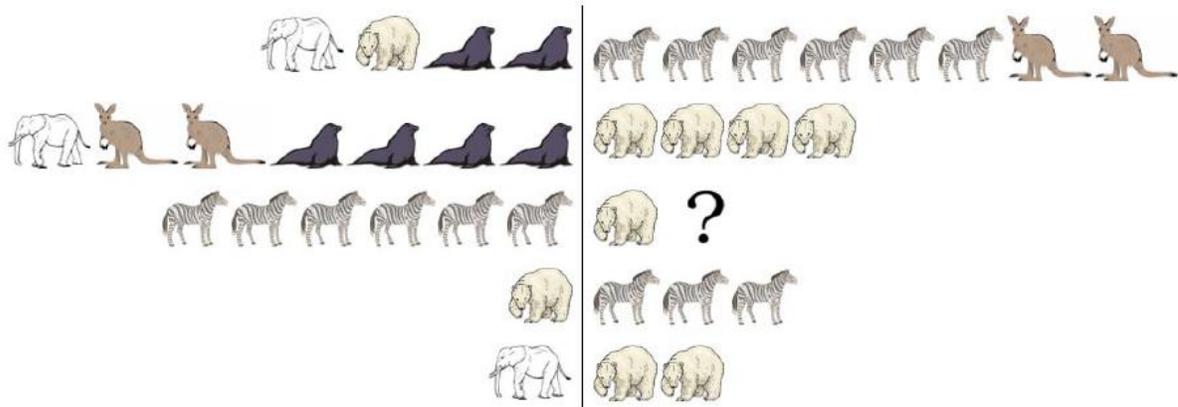
### Notes:

# Noah's Ark

*\*Credit to Fawn Nguyen*

## Problem:

Noah wants his ark to sail along on an even keel. The ark is divided down the middle, and on each deck the animals on the left exactly balance those on the right, except for the third deck. Can you figure out how many SEALS are needed in place of the question mark so that the ship balances?



## Extension:

Could you make up your own ark problem?

## Notes:

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## 6-Sided Die

---

*\*Credit to Michael Pruner*

### Problem:

Imagine a typical 6-sided die, and notice that the sum of opposite faces is always seven. The one is across from the six, the two is across from the five, and so on. Now imagine that you were making your own six-sided die that did not have this restriction. How many different dice could be made?



---

### Extension:

What about a 3-sided die or an 8-sided die?

What if a 6-sided die had 2 of the same numbers (1, 1, 2, 3, 4, 5) or 3 of the same numbers (1, 1, 1, 2, 3, 4)?

---

### Notes:

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# Cubes Tower

*\*Credit to Fawn Nguyen*

## Problem:

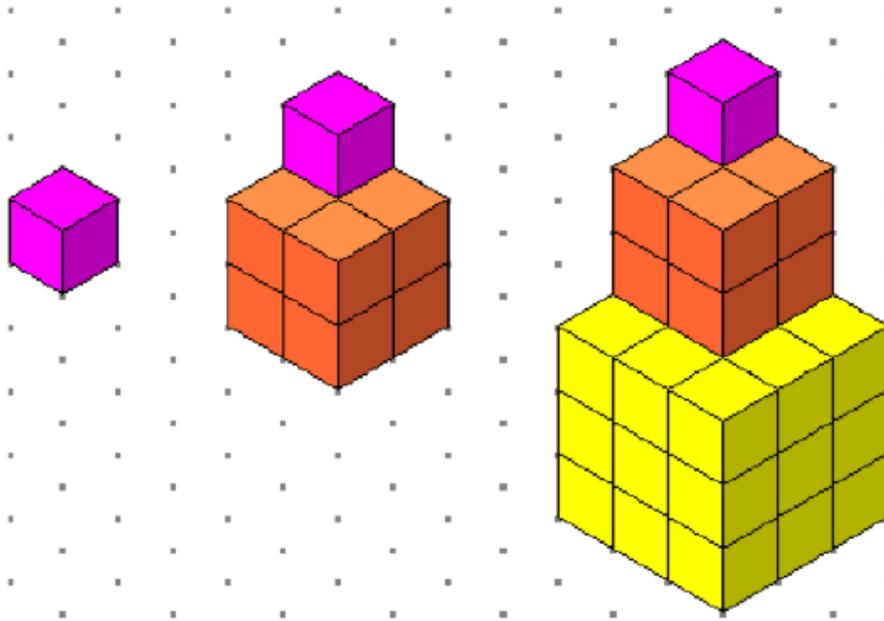


Figure 1

Figure 2

Figure 3

What do you notice? What do you wonder?

## Extension:

Sketch what Figure 4 might look like.

What would be an expression for this pattern?

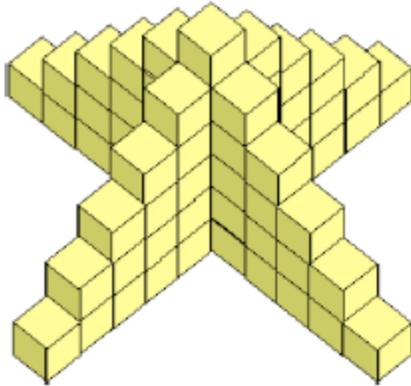
## Notes:

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## Another Tower

---

**Problem:**



How would we build the above cube tower with thirteen layers?

---

**Extension:**

---

**Notes:**

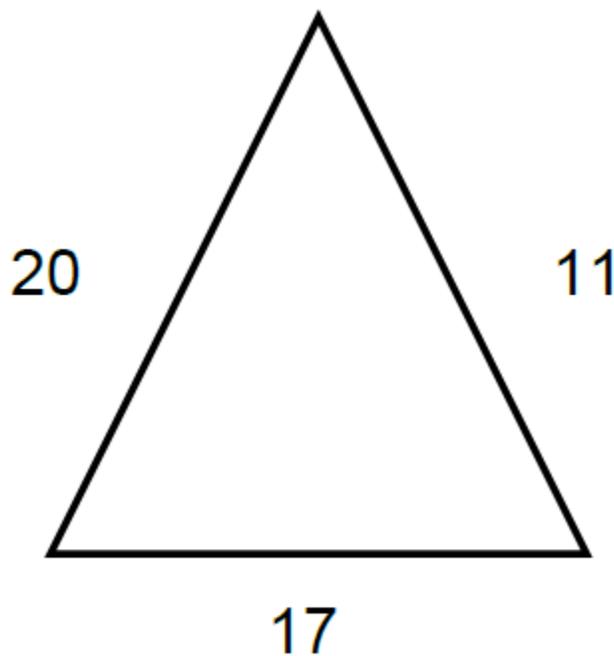
## Secret Numbers

---

*\*Credit to John Mason*

### Problem:

A secret number is assigned to each vertex of a triangle. On each side of the triangle is written the sum of the secret numbers at its ends. Explain a simple rule for revealing the secret numbers. An example has been given below.



---

### Extension:

Can you make your own Secret Numbers Puzzle?

---

### Notes:

[BACK](#)

## Pails

---

### Problem:

Three people carry five pails (each with capacity 8L) to a place where there are three springs. One of the springs gives 2 litres per minute and the other two give 1 litre per minute. It is not possible to use one spring to fill two pails simultaneously. It takes less than two minutes and more than one minute to take a pail from one spring to another. What is the shortest time it takes for them to fill all five pails? How is this accomplished?

---

### Extension:

---

### Notes:

## People at the Party

---

### Problem:

“How many people were at the party?” asked Carina’s mother.

“I do not know,” said Kadija’s mother, “but every two used a dish for noodles between them, every three used a dish for green onion cakes between them and every four used a dish for spring rolls between them.”

There were 65 dishes in all.

---

### Extension:

---

### Notes:

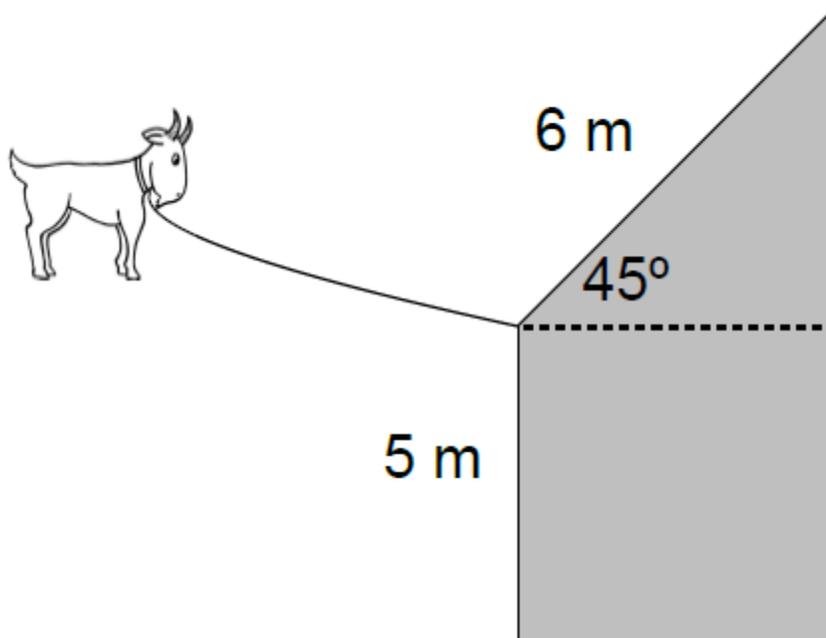
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## Grazing Goat

*\*Credit to John Mason*

### Problem:

A goat is tied with a rope to the corner of a shed as shown below (the diagram represents a view from above). If the rope's length when fully stretched is 7 metres, what is the maximum area that the goat can graze?



### Extension:

*\*Credit to John Mason*

Another Goat Problem - A goat is tethered by a  $6\text{ m}$  rope to the outside corner of a shed measuring  $4\text{ m}$  by  $5\text{ m}$  in a grassy field that needs to be mowed. What area of grass can the goat eat? What if the rope was fastened to the middle of one wall? What if the rope was  $20\text{ m}$  long? What if the shed was circular?

### Notes:

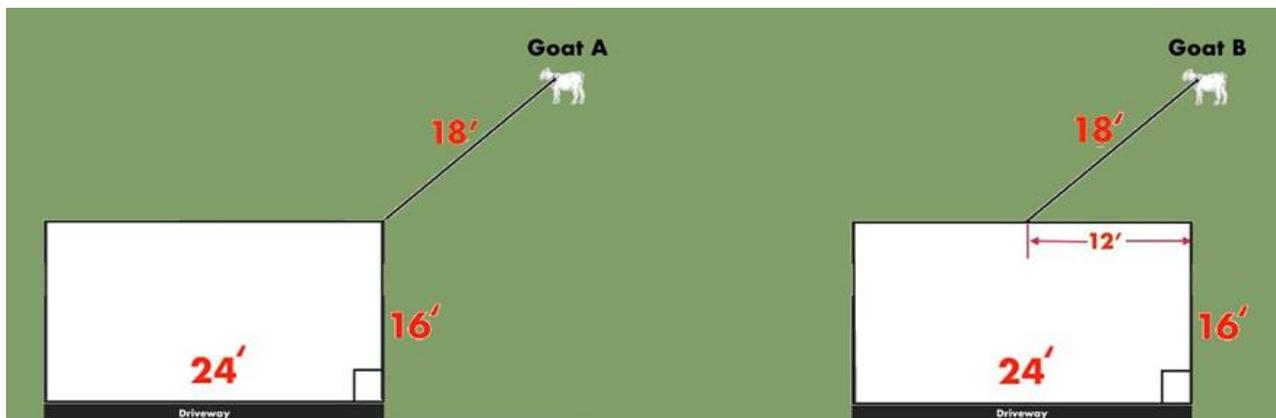
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## More Goats

---

### Problem:

Each goat is tethered by a chain at the point on the garage shown. Which goat has more grass to eat? How do you know?



---

### Extension:

Can you make your own goat problem?

---

### Notes:

## The Last Number

---

*\*Credit to Richard Hoshino*

### **Problem:**

Consider the string 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Cross out any two numbers in this list and add their difference as a new number at the end of the list. Continue the process of crossing out two numbers and adding the difference as a new number until only one number remains.

What can you say about the last number?

---

### **Extension:**

Once you have a conjecture, how are you going to check that it's true?

---

### **Notes:**

[BACK](#)

## Stack the Dice

---

*\*Credit to Peter Liljedahl*

### Problem:

Stack 5 dice one on top of each other. I will tell you the sum of all of the hidden faces.



---

### Extension:

What if you had more dice?

Make a tower of dice so all visible sides add up to 74.

---

### Notes:*\*for the teacher only\**

The sum of the hidden faces is the number of dice multiplied by 7 subtract the top face, so the above example is 33 (opposite faces on a die have a sum of 7).

[BACK](#)

# How Many Sevens?

---

*\*Credit to Peter Liljedahl*

## Problem:

If you write out the numbers from 1 to 1000, how many times will you write the number 7?

---

## Extension:

---

## Notes:

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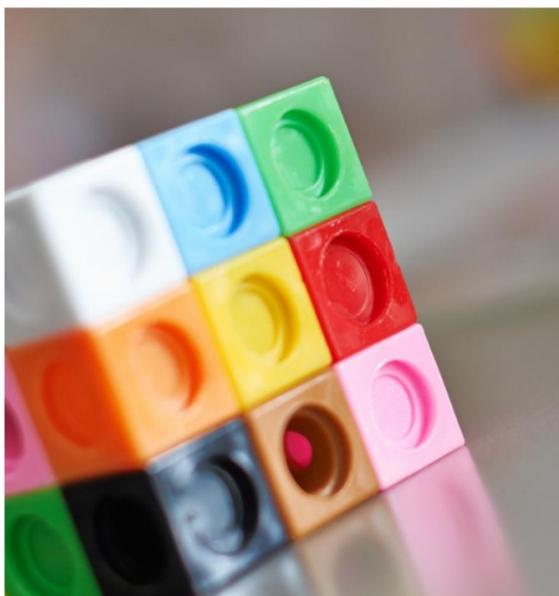
## Nine Colours

---

*\*Credit to nrich.maths.org*

### Problem:

## Nine Colours



You have 27 small cubes, 3 each of nine colours.

Can you use all the small cubes to make a 3 by 3 by 3 cube so that each face of the bigger cube contains one of each colour?

[nrich.maths.org](http://nrich.maths.org)

### Extension:

### Notes:

[BACK](#)

## Wolf, Sheep, and Cabbage

---

### Problem:

You need to move the wolf, sheep, and cabbage to the opposite shore by rowing them over one at a time in a boat. It gets more difficult though because when you are not around, the wolf will eat the sheep, the sheep will also do the same when alone with the poor little cabbage. How do you do this?



---

### Extension:

What if there was a different number of sheep, wolves, and cabbage?

---

### Notes:

[BACK](#)

## Ice Cream Shop

---

### Problem:

Your favourite ice-cream shop has 10 flavours of ice-cream.

How many different 2-scoop cones can you make with 10 flavours? What about 12 flavours?



---

### Extension:

What about  $N$  flavours?

What about 3-scoop cones?

---

### Notes:

Manipulatives are essential for this problem! Let the students pick which ones they would like to use.

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## Frog Jumping Puzzle

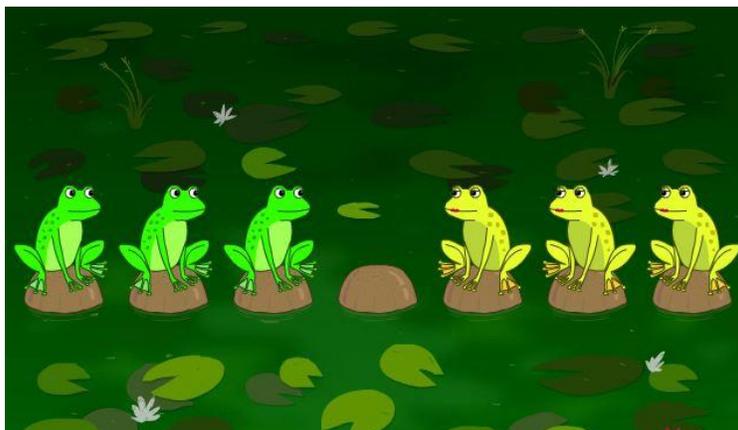
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\*Credit to <http://www.smart-kit.com/s7284/frog-jumping-puzzle/>

### Problem:

Can you get all of the dark green frogs to the right of the empty spot and all of the light green frogs to the left of the empty spot? The frogs move by either “hoping” or “sliding” forwards to a vacant spot.

**Note:** A frog can only slide forward into a vacant spot next to it OR can only hop forward over one other frog to land in an open spot.



---

### Extension:

What if there was a different number of frogs?

What about  $n$  frogs? Can you find a pattern? Can you graph your solution?

---

### Notes:

Don't forget the manipulatives!

For a digital exploration tool, see:

<http://www.smart-kit.com/s7284/frog-jumping-puzzle/>

For a sample solution and explanation, see:

[http://britton.disted.camosun.bc.ca/frog\\_puzzle\\_sol.htm](http://britton.disted.camosun.bc.ca/frog_puzzle_sol.htm)

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## More Jumping Frogs

---

*\*Credit to <https://math.stackexchange.com/questions/791182/frog-jump-problem>*

### Problem:

Lily pads are arranged in a circle. A frog starts jumping from one lily pad to the next. In the first jump, it skips one lily pad, in the next jump it skips two lily pads. In the third jump, it skips three, and so on. Can the frog reach all of the lily pads?



---

### Extension:

What numbers work? Why?

What numbers don't work? Why not?

---

### Notes:

[BACK](#)

## Hot Chocolate

---

### Problem:

How many different types of hot chocolate can you make if you have 5 ingredients but can only use 3 at a time?



---

### Extension:

What if there was a different number of ingredients per hot chocolate and/or a different number of ingredients to choose from?

Can you find any patterns?

---

### Notes:

Manipulatives are essential for this problem! Linking cubes work great. Different colours to represent the different ingredients. Kids can build towers to solve. Let them struggle first, then offer the cubes.

[BACK](#)

## Knights at a Round Table

---

### Problem:

A king sits at a round table. There are 3 empty seats. How many different ways can his 3 knights sit in them?

---

### Extension:

What if there were more spots at the round table?

What if there were  $n$  spots?

---

### Notes:

Another great problem for manipulatives!

[BACK](#)

## 6 Houses

---

*\*Credit to Henry Ernest Dudeney*

### Problem:

A circular road is 27 km long. On this road are six cottages, owned by 6 friends. The friends visit each other a lot, and they have noticed that every whole number from 1 to 26 (inclusive) is accounted for at least once when they calculate the distances from one cottage to another. Of course the friends can walk in either direction as required. Your task is to figure out where the houses are placed around the lake.



---

### Extension:

Can you find more than one solution?

---

### Notes:

Vertical non-permanent surfaces are great for this problem!

[BACK](#)

# Egg Drop

---

*\*Credit to <http://www.datagenetics.com/blog/july22012/index.html>*

## Problem:

You are given two eggs and access to a 100-storey building. How can you find out the highest floor from which an egg will not break when dropped out of a window?

If an egg is dropped and does not break, it is undamaged and can be dropped again. However, once an egg is broken, you may no longer use it. If an egg breaks when dropped from a floor, then it would also have broken from any floor above that. If an egg survives a fall, then it will survive any fall below that.

The question is: What strategy should you adopt to minimize the number of egg drops it takes to find the solution and what is the worst case for the number of drops it will take?

---

## Extension:

---

## Notes:

Sample solution: <http://www.datagenetics.com/blog/july22012/index.html>

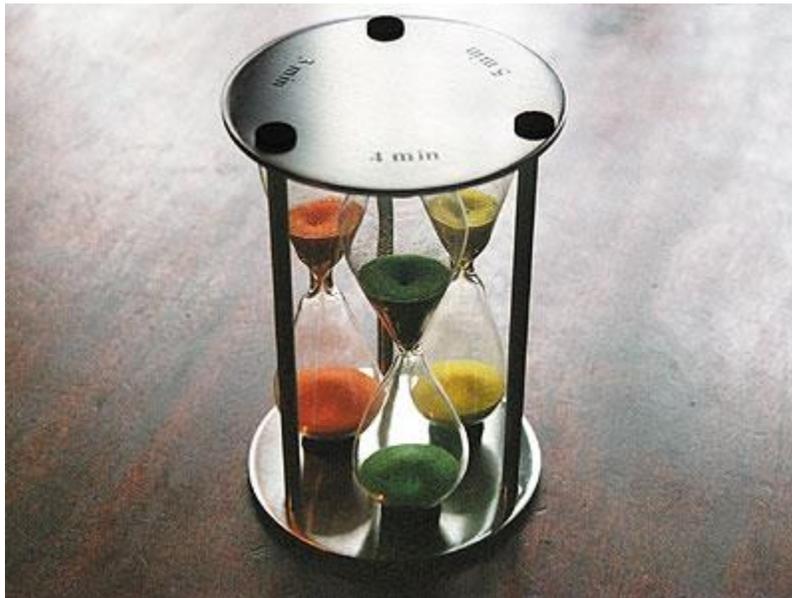
[BACK](#)

## Another Egg Timer

---

*\*Credit to <http://www.problempictures.co.uk/examples/op12.htm>*

### Problem:



When you turn over this egg timer, three sand timers measure 3 minutes, 4 minutes and 5 minutes. They cannot be turned over individually.

How could you use the timer to measure 6 minutes? 7 minutes?

Is it possible to measure 2 minutes? Is it possible to measure half a minute?

---

### Extension:

What other times could you measure?

What if you had three separate sand timers?

---

### Notes:

[BACK](#)

# Fancy Fence

---

*\*Credit to <http://www.problempictures.co.uk/examples/op09.htm>*

## Problem:



What do you notice? What do you wonder?

We know that the holes in the horizontal bar for the loops to pass through are 10cm apart.

---

## Extension:

How can you figure out how many loops you would need for a given length of railing?

How can you figure out what length of railing you would need for a given number of loops?

---

## Notes:

[BACK](#)

## Pile of Fruit

---

*\*Credit to <http://www.problempictures.co.uk/examples/op09.htm>*

### Problem:



What do you notice?

What do you wonder?

---

### Extension:

What size of pile could you build with a box of 200 oranges?

---

### Notes:

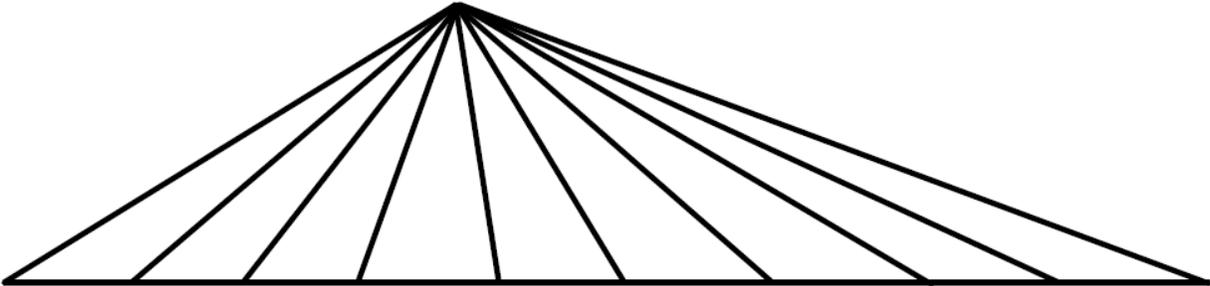
[BACK](#)

# How Many Triangles?

---

*\*Credit to Jim Matthews @Sienna College*

**Problem:**



---

**Extension:**

---

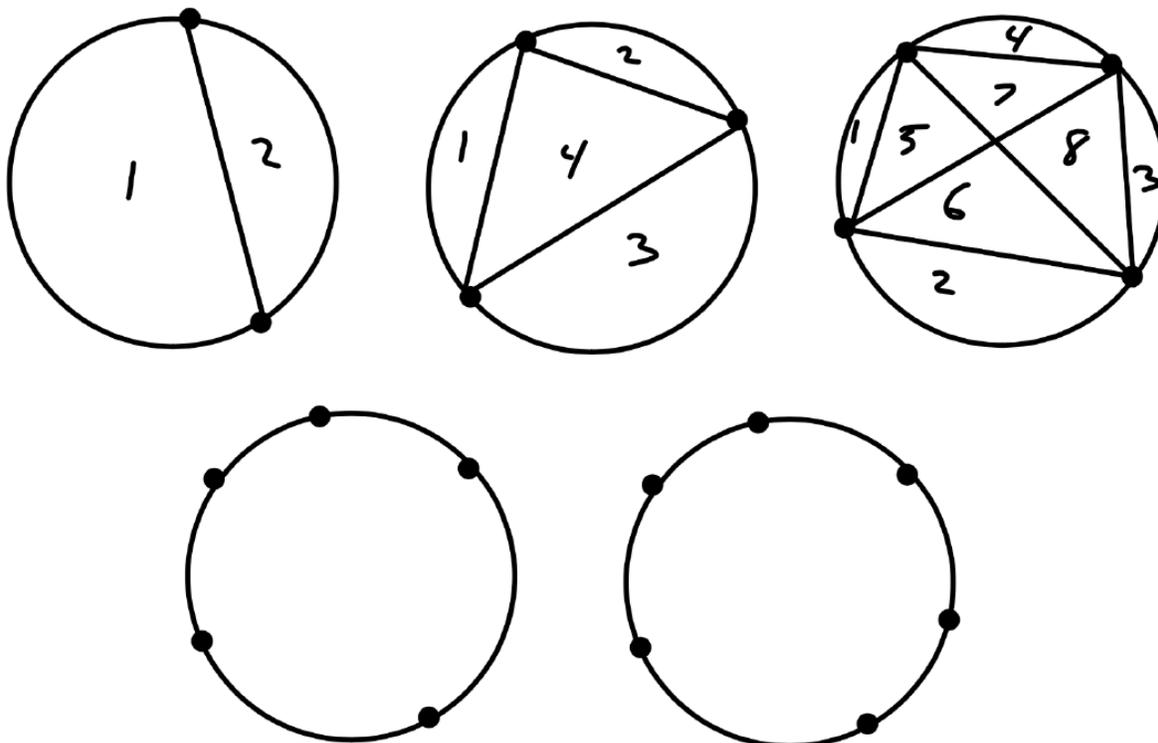
**Notes:**

## Points on a Circle

*\*Credit to Global Math Project*

### Problem:

How many regions do you get if you connect points on a circle?



### Extension:

### Notes:

Find the maximum number, so avoid too much symmetry!

[BACK](#)

## Garden Fence

---

### Problem:

You are building a rectangular garden. You have 40 meters of fencing. How can you make the area of the garden as large as possible?

---

### Extension:

---

### Notes:

## Exploring Area of Triangles

---

### Problem:

Draw a rectangle and then draw a triangle inside it with the base spanning the base of the rectangle. How much of the rectangle is covered by the triangle? Is this always the case?

---

### Extension:

Can you create a way to explain how to find the area of a triangle?

Can you show it more than one way?

Can you do it visually?

Can you create a formula?

---

### Notes:

Have a variety of paper, pencils, scissors, etc. available for students to use to explore. This experience should lead to the concept of area of a triangle (half the area of the rectangle,  $A=1/2 bh$ ).

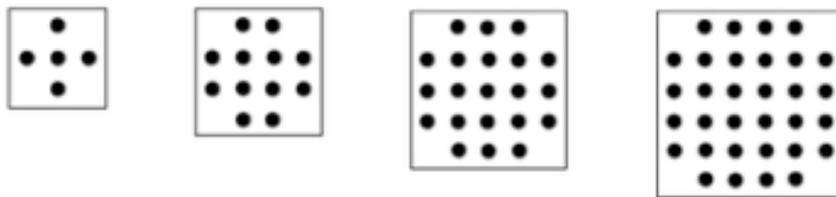
[BACK](#)

## Equivalent Expressions?

---

### Problem:

How can you use the visual pattern below to prove whether or not the following two expressions are equivalent?



$$(n + 2)^2 - 4 \quad \text{and} \quad n^2 + 4n.$$

---

### Extension:

Can you prove it algebraically? How many different ways?

---

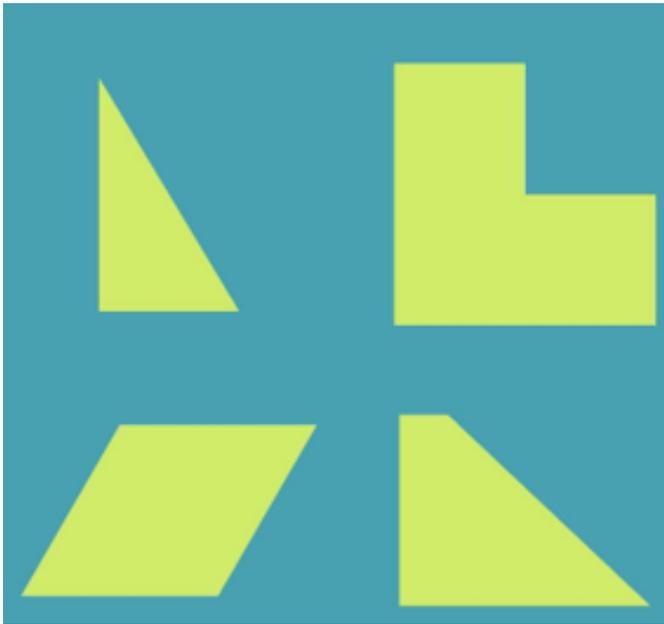
### Notes:

## Which One Doesn't Belong?

---

*\*Credit to <http://wodb.ca/>*

**Problem:**



Which one doesn't belong?

---

**Extension:**

How do you know?

Can you make your own?

---

**Notes:** Visit <http://wodb.ca/> for more puzzles!

[BACK](#)

## Estimation 180

---

*\*Credit to Andrew Stadel and Estimation 180*

### Problem:

How long is the song “Can’t Buy Me Love”?



### Extension:

What is an estimation that is too low? What is an estimation that is too high?  
What is an estimation that is just right? How do you know?

### Notes:

Video solution: <http://www.estimate180.com/day-127.html>

Visit: <http://www.estimate180.com/days.html> for more activities!

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# Visual Patterns - Frootloops

---

\*Credit to Fawn Nguyen and <http://www.visualpatterns.org/>

## Problem:

### *FROOT LOOPS*



Figure 1



Figure 2

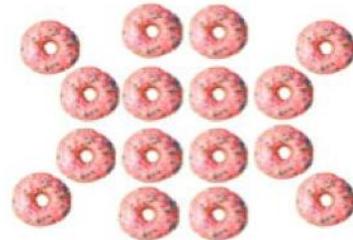


Figure 3

What do you notice? What do you wonder?

---

## Extension:

What would another figure look like?

Which figure has 324 fruit loops?

---

## Notes:

Visit: <http://www.visualpatterns.org/> for more visual patterns!

[BACK](#)

## Would You Rather – Credit Cards

\*Credit to John Stevens and <http://www.wouldyourathermath.com/>

### Problem:

# Have the Amazon Prime credit card OR the Capital One Quicksilver credit card?



**amazonPrime**  
D. BARRETT  
**VISA**  
Signature

- 5% back on all Amazon purchases
- 2% back at restaurants, gas stations, and drug stores
- 1% back on all other purchases
- \$0 annual fee (with \$100 Prime membership)
- 14.74% - 22.74% variable APR



**CapitalOne**  
**QUICKSILVER**

- Unlimited 1.5% cash back on all purchases
- \$100 cash bonus once you spend \$500 on purchases within the first 3 months
- \$0 annual fee
- 0% intro APR for 9 months;
- 13.49%-23.49% variable APR after that

[wouldyourathermath.com](http://www.wouldyourathermath.com)

### Extension:

### Notes:

Visit: <http://www.wouldyourathermath.com/> for more **Would You Rather** activities!

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## Related Percentages

---

Problem:

is 50% of   
and 75% of

---

Extension:

---

Notes:

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## Horse For Sale

---

### Problem:

You buy a horse for \$60, then sell it for \$70. You buy it back for \$80, and then sell it again for \$90. Did you make any money?

---

### Extension:

---

### Notes:

Visit <http://www.wouldyourathermath.com/> for more **Would You Rather** activities.

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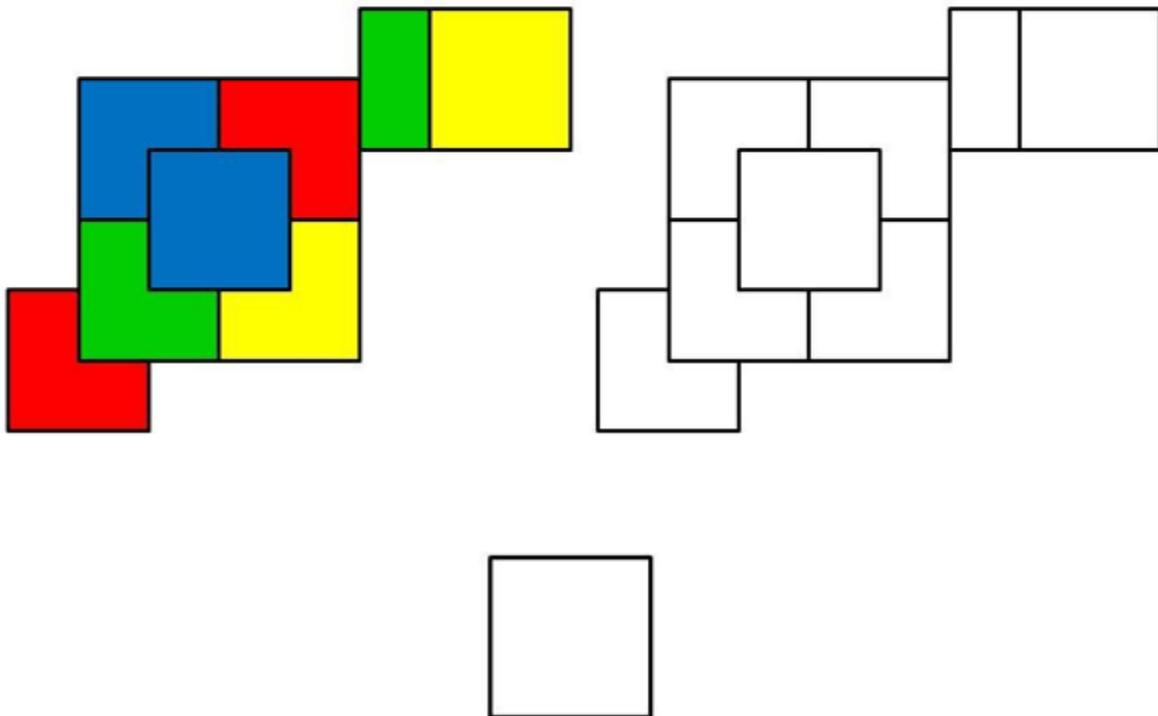
## Tiled Area

---

*\*Credit to Steve Wyborne and <http://www.stevewyborne.com/>*

**Problem:**

**What is the area of this shape?**



One square is one unit.

---

**Extension:**

---

**Notes:**

Visit <http://www.stevewyborne.com/?p=836> for more Tiled Area activities.

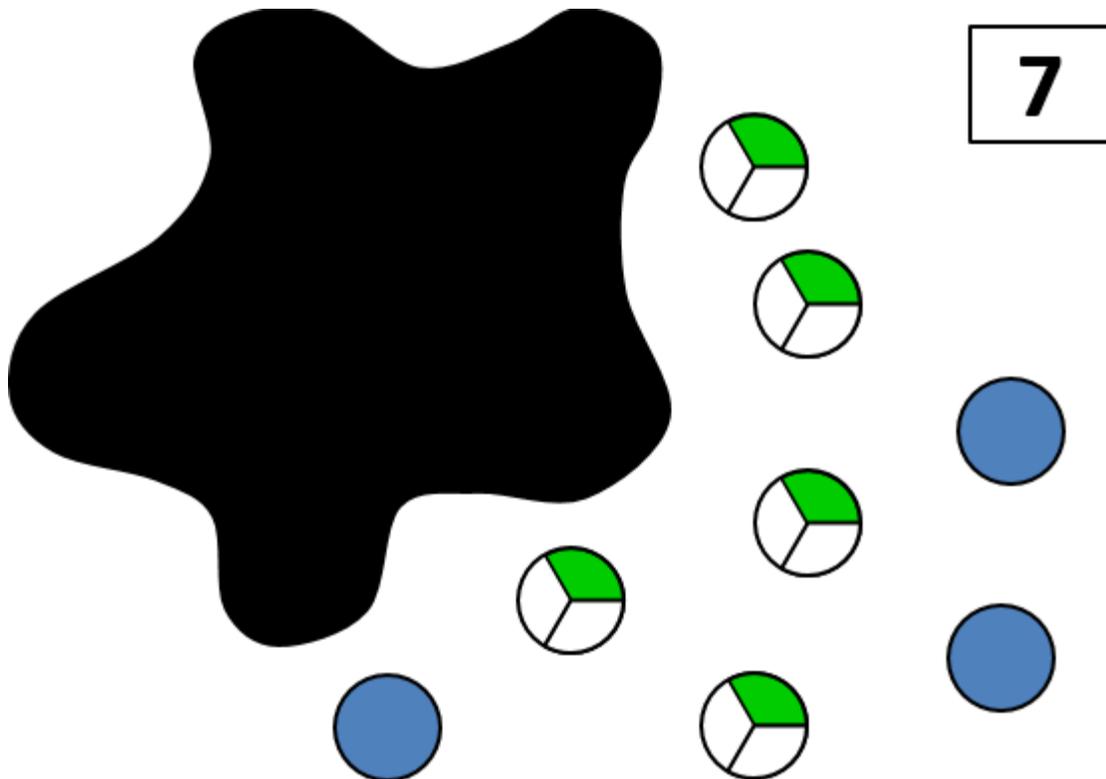
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# Fraction Splat!

*\*Credit to Steve Wyborne and <http://www.stevewyborne.com/>*

## Problem:

What number is under the splat?



## Extension:

What can we learn from this picture?

Can you make your own Fraction Splat!?

## Notes:

Visit <http://www.stevewyborne.com/?p=1028> for more Fraction Splat! activities.

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# Mixing Sodas

---

*\*Credit to Michael Pruner and BCAMT #weeklymathtasks*

## Problem:

Over the holiday season, I found two crazy soda flavours: Peppermint and Prune. I poured a tall glass of each and then decided to mix them. I carefully measured 30 mL of the Peppermint soda and poured it into the Prune soda glass. After stirring the Prune soda mix, I measured 30mL of this mix and poured it back into the Peppermint soda glass, creating a Peppermint soda mix. After all of this, which glass is more pure? In other words, is the Peppermint mix more pepperminty or is the Prune mix more pruney?

---

## Extension:

What if we introduced a third soda: Cabbage flavor?

---

## Notes:

Follow @BCAMT more #weeklymathtasks.

[BACK](#)

## Split 25

---

*\*Credit to <http://www.playwithyourmath.com/>*

### Problem:

Take the number 25, and break it up into as many pieces as you want.

$$25 = 10+10+5$$

$$25 = 2+23$$

$$25 = 1+1$$

What is the biggest product you can make if you multiply those pieces together?

---

### Extension:

What if it didn't have to be split into whole numbers?

What if the exponents didn't have to be whole numbers?

Will your strategy work for any number?

---

### Notes:

Visit <http://www.playwithyourmath.com/> for more Play With Your Math tasks.

[BACK](#)

# Perfect Numbers

---

*\*Credit to Sunil Singh*

## Problem:

6 is a perfect number because the sum of its factors (other than itself) is equal to itself. 8 is not a perfect number and neither is 12. Can you find other perfect numbers?



---

## Extension:

It is not known whether there are any odd perfect numbers...

---

**Notes:** Cuisenaire rods and linking cubes are both excellent manipulatives for this problem.

[BACK](#)

# Golf Ball Pyramid

---

**Problem:**



What do you notice? What do you wonder?

---

**Extension:**

---

**Notes:**

[BACK](#)

# Chicken Nuggets

---

\*Credit to <http://www.playwithyourmath.com/>

## Problem:

Chicken nuggets come in boxes of 6, 9, and 20.



6



9



20

What is the **largest number** of nuggets that you **CANNOT** buy when combining various boxes?

*I'll take 11 chicken nuggets, please!*

*I'm sorry, but that's not possible.*

---

## Extension:

Will your strategy work for any number?

---

## Notes:

Visit <http://www.playwithyourmath.com/> for more Play With Your Math tasks.

[BACK](#)

## Consecutive Sums

---

*\*Credit to <http://www.playwithyourmath.com/>*

### Problem:

Some numbers are sums of consecutive numbers.

Can you make all the numbers this way?

Which numbers can be written in more than one way?

---

### Extension:

In how many ways can a number  $n$  be written as the sum of two or more consecutive positive integers?

---

### Notes:

Visit <http://www.playwithyourmath.com/> for more Play With Your Math tasks.

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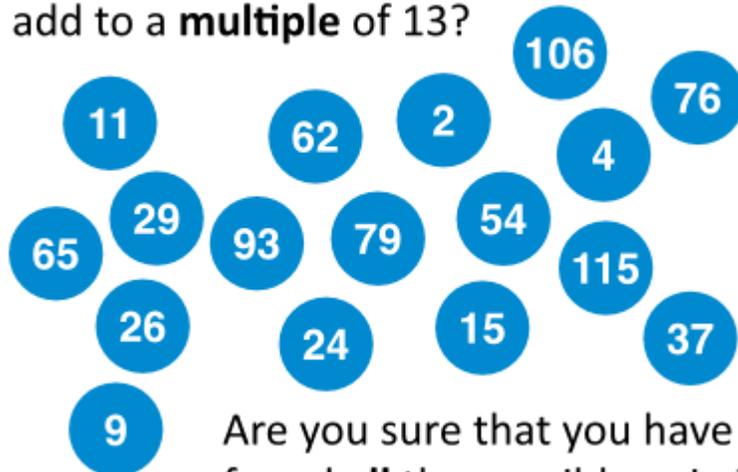
# Thirteens

---

\*Credit to <http://www.playwithyourmath.com/>

## Problem:

How many **pairs** of these numbers  
add to a **multiple** of 13?



---

## Extension:

---

## Notes:

Visit <http://www.playwithyourmath.com/> for more Play With Your Math tasks.

[BACK](#)

# Hailstone Sequences

---

*\*Credit to <http://www.playwithyourmath.com/>*

## Problem:

Hailstone Sequences follow these rules: Start with a number. If the number is even, divide it by 2. If the number is odd, multiply it by 3 and add 1.

What do you notice? What do you wonder?

---

## Extension:

Can you make a conjecture?

Is this true for every number?

This is considered an unsolved problem in mathematics.

---

## Notes:

This is considered an unsolved problem in mathematics. Collatz Conjecture (1937)

See <http://mathworld.wolfram.com/HailstoneNumber.html> for more information.

Visit <http://www.playwithyourmath.com/> for more Play With Your Math tasks.

[BACK](#)

# Passing Oranges

---

*\*Credit to Tim Bell at <http://csunplugged.org/>*

## Problem:

This is a co-operative problem solving game.

The aim is for each person to end up holding the 2 oranges labeled with their own letter.

1. Groups of five or more children sit in a circle.
2. The children are labeled with a letter of the alphabet (using name tags or stickers). There are two oranges with each child's letter on them, except for one child, who only has one corresponding orange to ensure that there is always an empty hand.
3. Distribute the oranges randomly to the children in the circle. Each child has two oranges, except for one child who has only one. (No child should have an orange with their letter on it.)
4. The children pass the oranges around until each child gets the oranges labeled with their letter of the alphabet. You must follow two rules:
  - a) Only one orange may be held in a hand.
  - b) An orange can only be passed to an empty hand of an immediate neighbour in the circle. (A child can pass either of their two oranges to their neighbour.)

---

## Extension:

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## Notes:

Visit <http://csunplugged.org/> for more computer science activities.

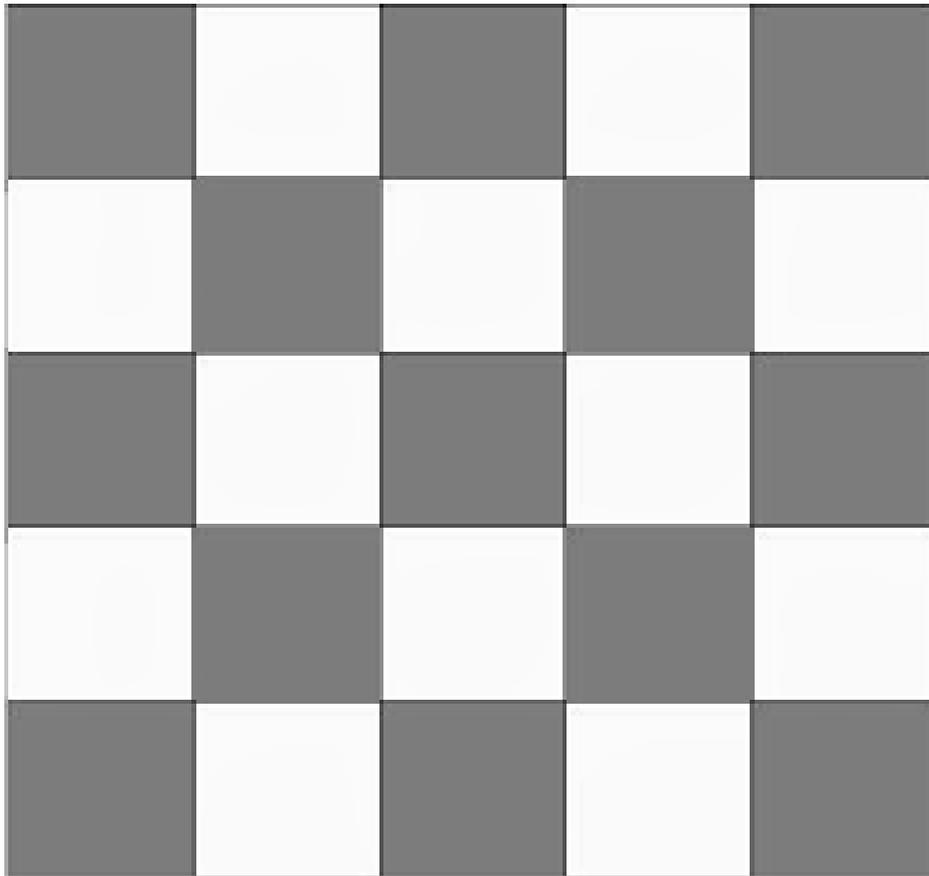
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# Wolves and Sheep

---

## Problem:

On a 5x5 chessboard, place 5 wolves (who can move like chess queens) and 3 sheep so that all the sheep are safe from being eaten by the wolves.



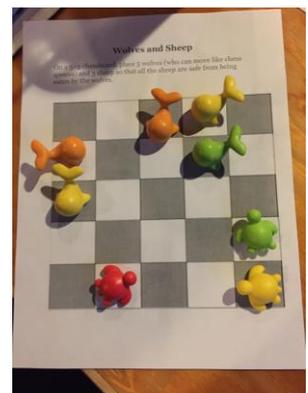
## Extension:

What if it was a different size chessboard?

**Notes:** Provide manipulatives and copies of chessboard.

\*Teacher Only\* - See Sample Solution

This is challenging!



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# Dragon Curve Fractal

---

## Problem:

Imagine a long strip of paper folded in the same direction once, twice, and then a third time. Then you unfold it...

What do you notice?

What do you wonder?

What do you want to investigate?

What if you keep folding?

---

## Extension:

When the paper is finally unfolded, and the creases made to equal  $90^\circ$ , what do you notice?

Explore, investigate, and create a Fractal (with a computer program or by hand, draw or build a representation)!

Examples: Koch Snowflake, Sierpinski Triangle, Jerusalem Cube

---

## Notes:

\*I love this Dragon Curve video by Pierre Bernard [https://youtu.be/3WBvS\\_n2oTY](https://youtu.be/3WBvS_n2oTY)

\*Another favourite by Vi Hart <http://www.youtube.com/watch?v=dsvLLKQCxeA>

# Pumpkins

---

\*Credit to Vector 51(1) – p. 41: <https://tinyurl.com/k3q6rsy> and BCAMT

## Problem:

A 600-pound pumpkin was entered in a contest. When it arrived, it was 99% water. The pumpkin sat for days in the hot sun, lost some weight (water only), and is now 98.5% water. How much does it now weigh?

---

## Extension:

At 98.5%, the pumpkin had lost 0.5% water. What if the pumpkin loses 1%, 2%, n%?

---

## Notes:

[BACK](#)

## 25 Coins

---

*\*Credit to John Mason – Thinking Mathematically*

### **Problem:**

25 coins are arranged in a 5 by 5 array. A fly lands on one, and tries to hop onto every coin exactly once, at each stage moving only to the adjacent coin in the same row or column. Is this possible?

---

### **Extension:**

Can you explain why some starting locations are not possible?

What if there were 4 dimensions?

What about rectangles?

---

### **Notes:**

[BACK](#)

# Pancakes

---

*\*Credit to John Mason – [Thinking Mathematically](#)*

## Problem:

When I make pancakes, they all come out different sizes, I pile them up on a plate in the warming oven as they are cooked, but to serve them attractively, I would like to arrange them in order with the smallest on top. The only sensible move is to flip over the topmost ones. Can I repeat this sort of move and get them all in order?

---

## Extension:

What is the most complicated arrangement for 3, 4, or 5 pancakes? What is the minimum number of flips for these arrangements?

---

## Notes:

This is an open mathematics problem.

See [https://en.wikipedia.org/wiki/Pancake\\_sorting](https://en.wikipedia.org/wiki/Pancake_sorting)

[BACK](#)

## Box of Marbles

---

\*Credit to Vector 55(1) – p.49: <https://tinyurl.com/zaksq6b> and @BCAMT

### Problem:

In a box, you have 13 white marbles and 15 black marbles. You also have 28 black marbles outside the box. Remove two marbles, randomly, from the box. If they are of different colours, put the white one back in the box. If they are the same colour, take them out and put a black marble back in the box. Continue this until only one marble remains in the box. What colour is the last marble?

---

### Extension:

---

### Notes:

Marbles or manipulatives.

[BACK](#)

# Sharing Bacon

---

*\*Credit to John Mason – [Thinking Mathematically](#) and @BCAMT*

## Problem:

You are a chef at a summer camp and you are frying 30 identical strips of bacon for breakfast. A counselor comes in to inform you that there are only 18 campers coming in for breakfast and they all love bacon. How are you going to equally share the bacon between the campers?

What is the minimum number of cuts necessary? How do you know?

What is the minimum number of pieces? How do you know?

---

## Extension:

What about sharing amongst 17 campers? 16 campers?  $N$  campers?

---

## Notes:

Manipulatives, bacon?

[BACK](#)

# Square Peg in a Round Hole

---

*\*Credit to Peter Liljedahl*

## **Problem:**

What fits better: A square peg in a round hole or a round peg in a square hole?

---

## **Extension:**

Want to explore a different shaped plug in a different shaped whole?

---

## **Notes:**

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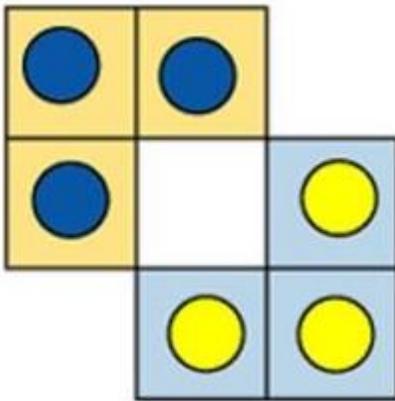
## Switch Positions

---

\*Credit to [www.mathfair.com/other-puzzles.html](http://www.mathfair.com/other-puzzles.html)

### Problem:

The picture below show a 3 by 3 board with two missing corners. There are 3 blue chips on the yellow part of the board and 3 yellow chips on the blue part of the board.



The task is to put the blue chips on the blue part and the yellow chips on the yellow part. The chips can only move horizontally or vertically into an empty space or they may leapfrog a single chip into an empty space.

---

### Extension:

What is the minimum number of moves? How do you know? What if you expand the board?

---

### Notes:

Copy of the board, two colours of manipulatives?

# Magic Squares

---

**Problem:**


Place the numbers 1 through 9 so that each row, column, and diagonal adds up to the same number (use each number exactly once). Find all possible arrangements. Prove that there are no other ways.

Can you come up with a 2x2 magic square? What about a 4x4 magic square?

---

**Extension:**

What value does each row, column, and long diagonal need to sum to in a  $n \times n$  magic square?

Investigate Magic Rectangles and Magic Triangles.

---

**Notes:**

See video for a sample solutions <https://youtu.be/zPnN046OM34>

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## Milk Crate

---

*\*Credit to John Mason – Thinking Mathematically*

### **Problem:**

A certain milk crate can hold 36 bottles of milk. Can you arrange 14 bottles in the crate so that each row and column has an even number of bottles?

---

### **Extension:**

What is the smallest array that can fit 14 bottles under this rule? What about 15 bottles?

---

### **Notes:**

## Factor Craze

---

**Problem:**

Prime numbers have exactly two factors – 1 and itself. Which numbers have exactly 3 factors? Exactly 4 factors?

---

**Extension:**

Given any positive integer, how can you tell exactly how many factors it has?

---

**Notes:**

# Hotel Rooms

---

*\*Credit to Peter Liljedahl and @BCAMT*

## Problem:

There is a hotel with  $n$  rooms in a row. Every room has a door to the corridor and doors connecting it to the adjacent rooms. A woman has rented all  $n$  rooms, telling the hotel manager that if he needs to contact her, she will always be in a room adjacent to the room she was in the day before. The hotel manager needs to contact the woman regarding an issue with her credit card; however, he is so busy that he only has time to check one room each day. Can you devise a scheme such that the hotel manager is guaranteed to find the woman or is she going to stay for free?

---

## Extension:

---

## Notes:

Start this task with 2 rooms, then 3 rooms, and when groups have convincing well-reasoned solutions, move to 4 rooms, 5 rooms, etc.

Many people like to use a table or a chart to help solve this problem.

***\*Great enrichment problem for an extra challenge - It can be quite difficult!***

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## Exploding Dots - Add, Multiply, Subtract, Divide, Decimals

---

*\*Credit to James Tanton*

### Problem:

*“When I was a young child I invented a machine (not true) that was nothing more than a series of boxes that could hold dots. And these dots would, upon certain actions, explode. And with this machine, I realized I could explain true things! In one fell swoop I explained all the mathematics of arithmetic I learnt in grade school (true), all of the polynomial algebra I was to learn in high-school (true), elements of calculus and number theory I was to learn in university (true), and begin to explore unanswered research questions intriguing mathematicians to this day (also true)!”*

*–James Tanton*

Here is the video that explains all of Exploding Dots at once:

<https://vimeo.com/204368634>

Here is the very first video lesson from James Tanton:

<https://youtu.be/vstloER3idM>

Here is a kid video to get things started with your students:

<https://youtu.be/cwicTRuLT4Y>

Here is the link to the Exploding dot “experiences” that you can do with your students (Adding, Multiplication, Subtraction, Division, Decimals):

<http://gdaymath.com/courses/exploding-dots/>

---

### Notes:

Manipulatives and chalk, whiteboards and markers, time to explore, understand, and explain in groups! Watch the first video first to get a feel for it as a teacher. Dive in with your students as you work through parts of the video or the “experiences” (search YouTube for **James Tanton Exploding Dots** for the different topics). Take risks and have fun! Once you are done working with the basic operations, decimals comes naturally.

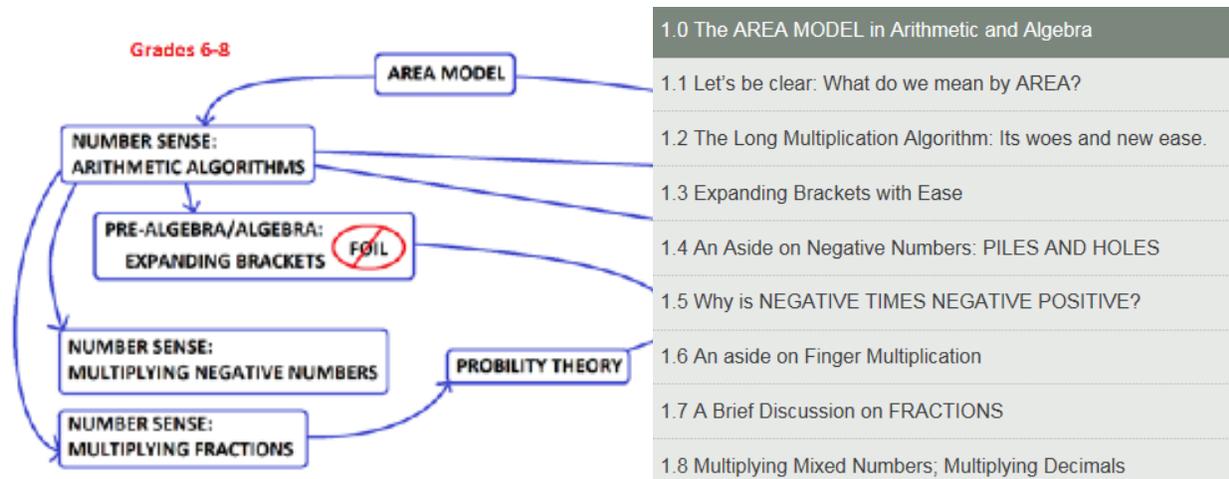
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## Area Models – Multiplication, Integers, Fractions, Decimals

*\*Credit to James Tanton*

### Problem:

How to learn...



Here is the link to the Area Model “experiences” that you can do with your students (Multiplication, Integers, Fractions, Decimals):

<http://gdaymath.com/courses/astounding-power-of-area/>

### Notes:

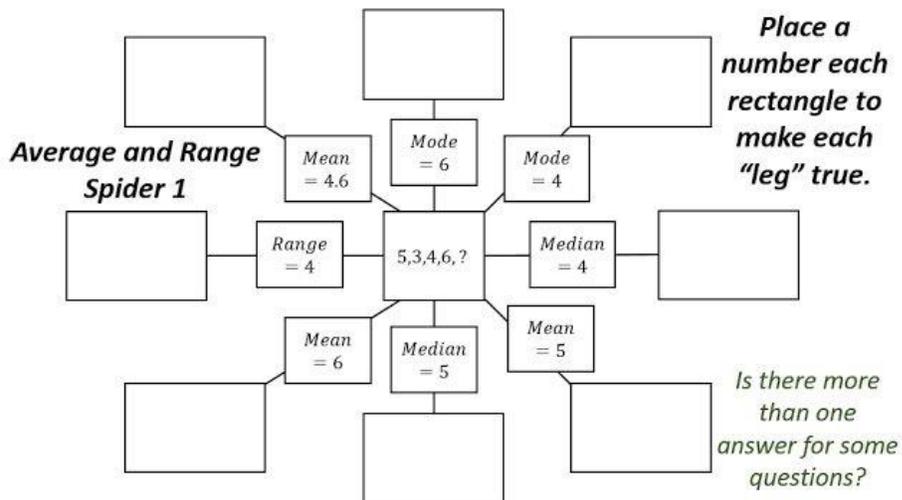
Whiteboards and markers, time to explore, understand, and explain in groups! Dive in with your students as you work through the “experiences”. Take risks and have fun!

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# Spider – Mean, Mode, Median, Range

\*Credit to Sarah Carter and @mathequalslove

## Problem:



## Extension:

Is there more than one answer?

Can you make your own Spider puzzle?

## Notes:

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## Fraction Talks – Design Tasks

\*Credit to Nat Banting and his blog <http://musingmathematically.blogspot.ca/2015/06/fraction-talks.html>

### Problem:

Using only three colours of square tiles can you build a shape where...

at least  $\frac{1}{2}$  is Red  
 at least  $\frac{1}{4}$  is Blue  
 no more than  $\frac{1}{6}$  is Yellow



at least  $\frac{1}{4}$  is Red  
 at least  $\frac{1}{12}$  is Blue  
 the number of Yellow squares are more than double than the number of Red squares

more than  $\frac{1}{2}$  is Red  
 exactly half the number of Red squares are Blue  
 the remaining is Yellow

How do you know? How can you prove it? Is there another way?

### Extension:

If you use a total of 100 squares, how many ways can you meet all requirements?  
 Can you make your own Design Task?

### Notes:

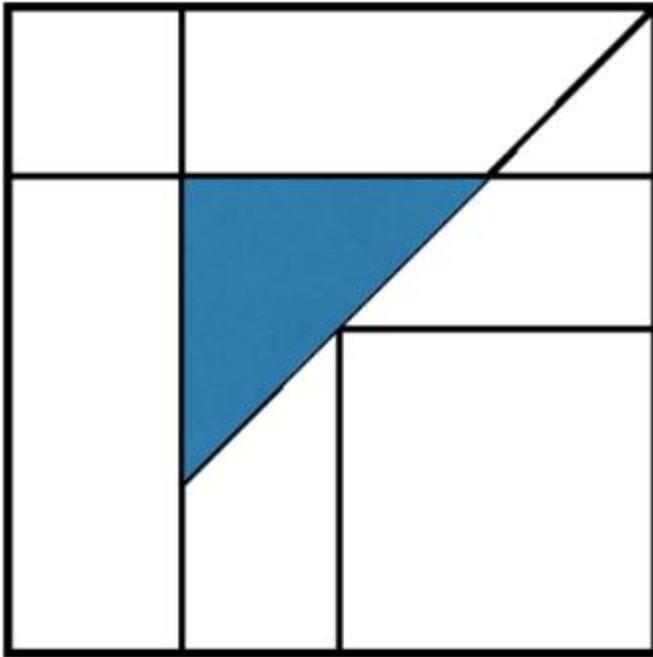
Visit [www.fractiontalks.com](http://www.fractiontalks.com) and <https://drive.google.com/file/d/0B9hruAPIqvU5ZIFfMDJvUU5udzA/view> to learn more about Design Tasks and Fraction Talks

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## Fraction Talks

\*Credit to Nat Banting and his blog <http://musingmathematically.blogspot.ca/2015/06/fraction-talks.html>

### Problem:



What fraction of the square is shaded? How do you know?  
 Can you shade a section that is exactly double the blue? How do you know?

### Extension:

See 12 Days of Fraction Talks

<https://drive.google.com/file/d/0B9hruAPlgvU5ZC1OOUFoZ0RZNUE/view>

### Notes:

Visit <http://www.fractiontalks.com/> to learn more about Fraction Talks.

Read more <https://drive.google.com/file/d/0B9hruAPlgvU5ZIFfMDJvUU5udzA/view>

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## 36 Pop Cans

---

*\*Credit to Alex Overwijk @alexoverwijk*

### Problem:

How should we package 36 pop cans?

What assumptions did you make?

What math did you consider?



---

### Extension:

What about 1000 cans?

<http://time.com/money/4761382/finnish-brewery-nokina-panimo-1000-cans-keisari-lager/>



---

### Notes:

Can has a height of 12cm and a diameter of 6.5cm. Best Arrangement?  
Dimensions of the box? Surface Area of the box? Cost of the box? Volume of the dead space?

[BACK](#)

# Packaging Golf Balls

---

*\*Credit to Alex Overwijk @alexoverwijk*

## Problem:

How would you package a dozen golf balls?



What is the optimal arrangement of the balls?

What is the optimal shape of the box? Cost? Waste? Size?

---

## Extension:

How many ways could you package 96 golf balls?

What is the best way?

What else would be interesting to package?

---

## Notes:

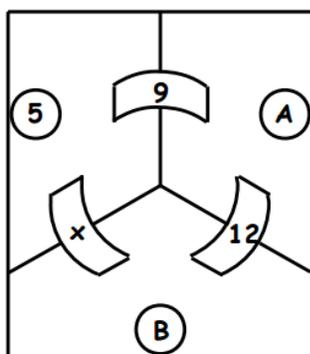
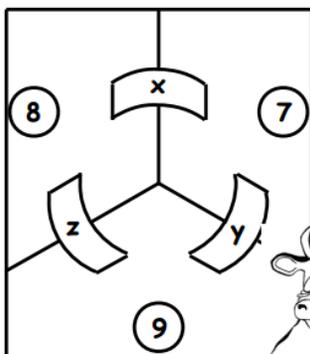
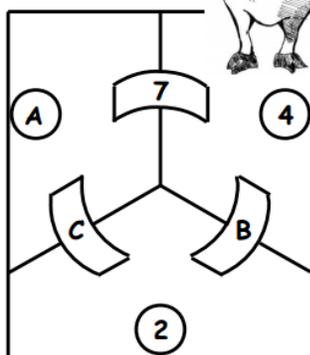
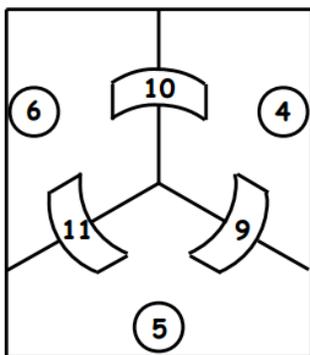
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## Cows in the Classroom

\*Credit to <http://www.bovinemath.com/>

### Problem:

In the following diagrams, the number on each bridge is the **sum** of the numbers of cows in each of the adjoining fields. Pasture one has been completed. Find all of the unknown values. I.#1



**Extension:** Visit <http://www.bovinemath.com/> for more complex puzzles.

**Notes:** Visit <http://www.bovinemath.com/> for more.

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## Migrating Cows

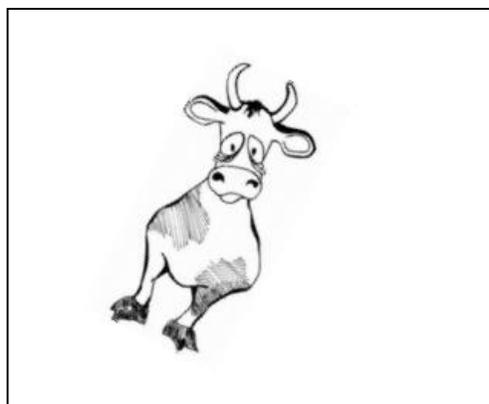
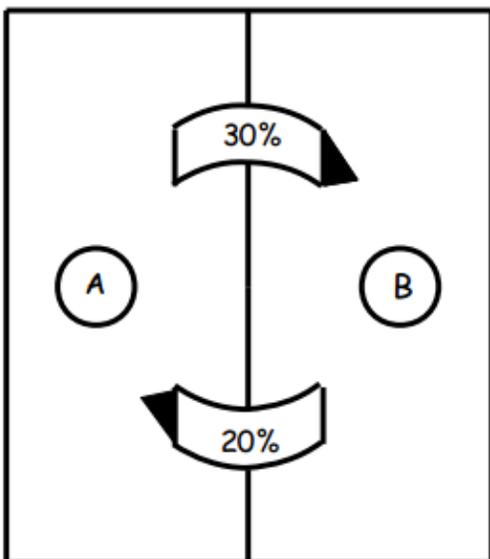
\*Credit to <http://www.bovinemath.com/>

### Problem:

In the diagram below, the percentages and arrow on each bridge indicate the daily rate of migration of cows from one field to another.

What is the smallest total number of cows that would make this puzzle possible?

Is it possible that the number of cows in each field doesn't change with migration? What is the smallest total number of cows that would make this happen?



### Extension:

If  $A=90$  and  $B=10$ , what would happen to the distribution of cows after several days of migration?

What if you started with different numbers?

What do you notice? Can you draw any conclusions?

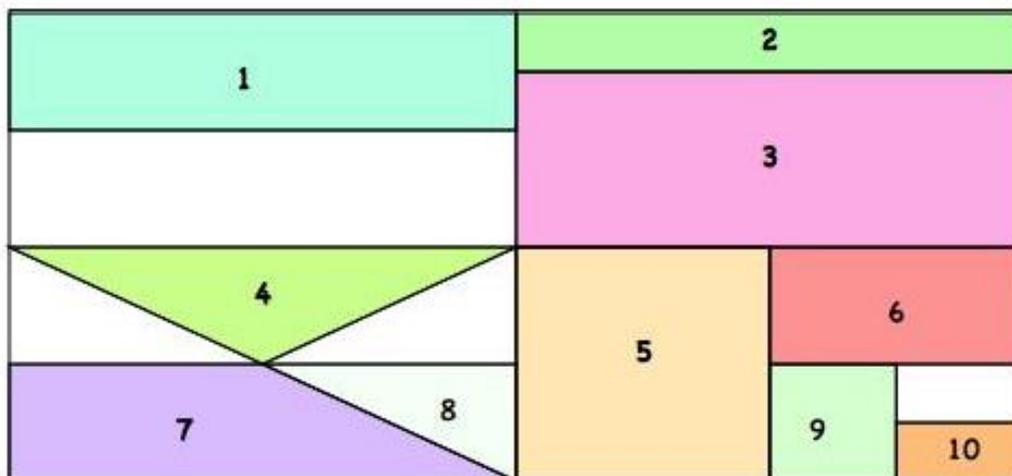
**Notes:** Visit <http://www.bovinemath.com/> for more.

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# Rectangle Tangle

*\*Credit to NRICH*

## Problem:



The large rectangle above is divided into a series of smaller quadrilaterals and triangles. Each of the shapes is a fractional part of the large rectangle.

Can you untangle what fractional part is represented by each of the ten numbered shapes?

## Extension:

Can you make your own Rectangle Tangle?

## Notes:

Visit <https://nrich.maths.org/1048>

[BACK](#)

## More Cows

---

Here's a word problem that has been attributed to Sir Isaac Newton! \*Credit to [www.bowvinemath.com](http://www.bowvinemath.com)

### Problem:

Three cows eat all the grass on two acres of land, together with all the grass that grows there in two weeks. Two cows eat all the grass on two acres of land, together with all the grass that grows there in four weeks. How many cows, then, will eat all the grass on six acres of land together with all the grass that grows there in the six weeks?

---

### Extension:

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**Notes:** Visit <http://www.bovinemath.com/> for more.

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# Equato

*\*Credit to Gregory Tang*

## Problem:

Fill in the empty boxes to make every horizontal and vertical equation correct. Use the correct order of operations and read left to right and top to bottom. Use every number in the number bank once.

	1	2	3	4	5	6	7
7	+	3	-	8	=	2	
+		+		=		-	
6	+		-	9	=		
-		-		+		=	
	+	5	-		=	4	
=		=		-		-	
8	+		-		=		

## Extension:

Notes: Visit <http://gregtangmath.com/games>

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## TED-Ed “Can You Solve This Riddle” Videos

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*\*Credit to Ted-Ed and a variety of contributors*

**Problem:** Can you solve these riddles? Here is a link to a TED-Ed playlist with riddle videos. The first part of the video presents the riddle and the second part explains a solution. Make sure you present the riddle then stop if before the solution plays.

[https://www.youtube.com/playlist?list=PLJicmE8fK0EiFRt1Hm5a\\_7SJFaikIFW30](https://www.youtube.com/playlist?list=PLJicmE8fK0EiFRt1Hm5a_7SJFaikIFW30)

### Try these ones first:

Can you solve the passcode riddle? - Ganesh Pai

[https://www.youtube.com/watch?v=7Vd1dTBVbFg&list=PLJicmE8fK0EiFRt1Hm5a\\_7SJFaikIFW30&index=7](https://www.youtube.com/watch?v=7Vd1dTBVbFg&list=PLJicmE8fK0EiFRt1Hm5a_7SJFaikIFW30&index=7)

Can you solve the river crossing riddle? - Lisa Winer

[https://www.youtube.com/watch?v=ADR7dUoVh\\_c&list=PLJicmE8fK0EiFRt1Hm5a\\_7SJFaikIFW30&index=13&t=37s](https://www.youtube.com/watch?v=ADR7dUoVh_c&list=PLJicmE8fK0EiFRt1Hm5a_7SJFaikIFW30&index=13&t=37s)

Can you solve "Einstein's Riddle"? - Dan Van der Vieren

[https://www.youtube.com/watch?v=1rDVz\\_Fb6HQ&list=PLJicmE8fK0EiFRt1Hm5a\\_7SJFaikIFW30&index=10](https://www.youtube.com/watch?v=1rDVz_Fb6HQ&list=PLJicmE8fK0EiFRt1Hm5a_7SJFaikIFW30&index=10)

Can you solve the prisoner boxes riddle? - Yossi Elran

[https://www.youtube.com/watch?v=vldStMTgNI0&list=PLJicmE8fK0EiFRt1Hm5a\\_7SJFaikIFW30&index=6](https://www.youtube.com/watch?v=vldStMTgNI0&list=PLJicmE8fK0EiFRt1Hm5a_7SJFaikIFW30&index=6)

---

### Extension:

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**Notes:** Have manipulatives ready, as well as whiteboards and markers. Great problems to do without the teacher knowing the solution in advance. Model risk-taking, problem solving, collaboration, perseverance! It's ok if we don't always find the solution; we can still appreciate the beauty of the problem.

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## Six Cats and Six Rats

---

*\*Credit to Lewis Carroll*

### Problem:

If six cats can kill six rats in six minutes, how many cats are needed to kill 100 rats in 50 minutes?

---

### Extension:

If three hens lay four eggs in five days, how many days will it take a dozen hens to lay four dozen eggs?

---

### Notes:

[BACK](#)

# Stacking Cups

---

*\*Credit to David Sladkey @dsladkey and Andrew Stadel @mr\_stadel*

## Problem:

You have a stack of 2 red Solo cups (one inside the other), a stack of 2 Styrofoam cups (one inside the other), and a ruler. How could you predict the number of cups to make the heights of the 2 stacks the same?

---

## Extension:

How many red Solo cups would you need in a stack to reach the height of your teacher?

How many Styrofoam cups would you need?

What if you were stacking the cups in a pyramid?

---

## Notes:

Red Solo cups, Styrofoam cups, rulers, graph paper.

[BACK](#)

## Sums of 100%

---

### Problem:

Use the values in the box to make sums that equal 100%. How many different sums can you come up with? You may only use a number once in any one sum, but you may use that number again in a different sum.

68%	$\frac{1}{2}$	60%	$\frac{1}{20}$	1%	0.9	0.04	$\frac{13}{20}$	$\frac{1}{7}$
$\frac{3}{25}$	16%	55%	0.02	$\frac{1}{9}$	0.15	$\frac{1}{8}$	24%	0.44
0.06	$\frac{1}{6}$	$\frac{1}{4}$	29%	0.037	$\frac{1}{3}$	19%	$\frac{17}{50}$	

---

### Extension:

What if you could use any operation or combination of operations?

---

### Notes:

## Wordless Geometry

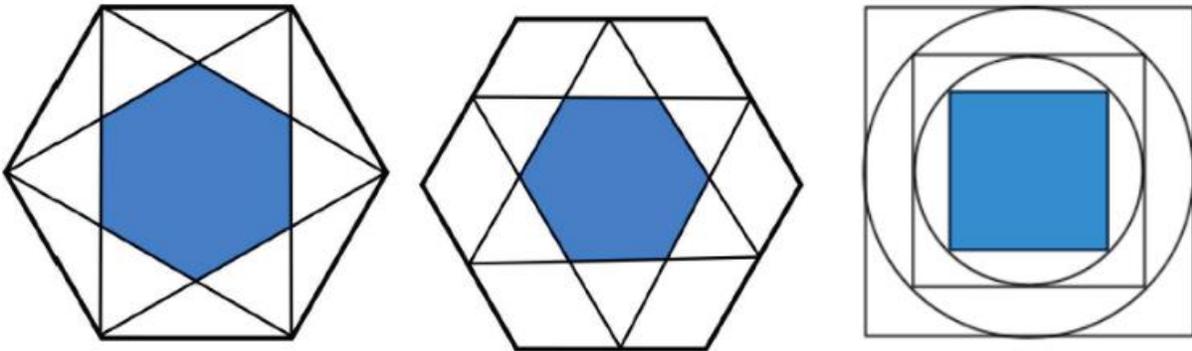
\*Credit to <http://www.celebrationofmind.org/?p=1060>

### Problem:

For every image, the question is the same: what fraction of the shape is shaded?

Each problem can be solved without numbers or words; just geometric observations.

You can assume that drawings are to scale, and that points that look like midpoints are midpoints, shapes that look like circles are circles, squares are squares, polygons are regular, etc.



**What fraction of each shape is shaded?**

### Extension:

Can you make your own?

### Notes:

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## More Wordless Geometry

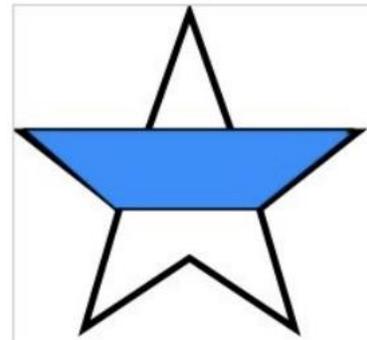
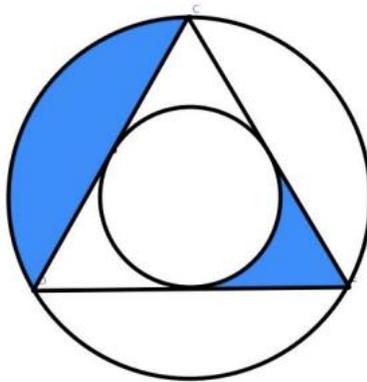
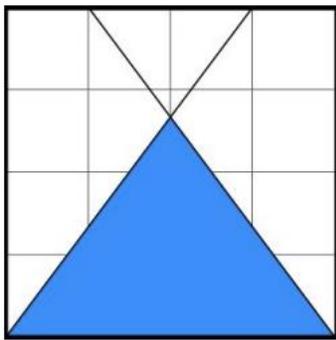
\*Credit to <http://www.celebrationofmind.org/?p=1060>

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**What fraction of each shape is shaded?**

### Extension:

Can you make your own?

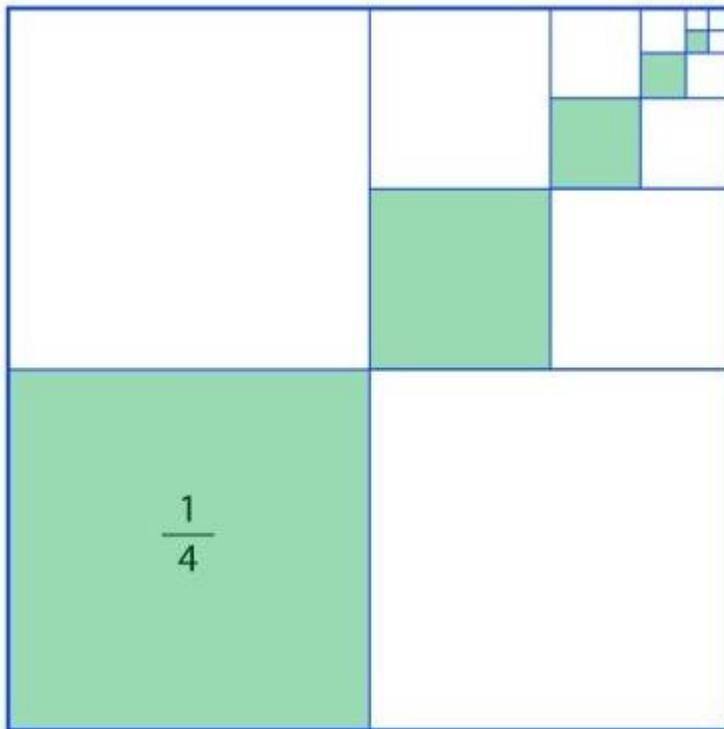
### Notes:

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## Part of a Whole

---

**Problem:**



What fraction of the whole square does the shaded part represent?

---

**Extension:**

What if the pattern continued?

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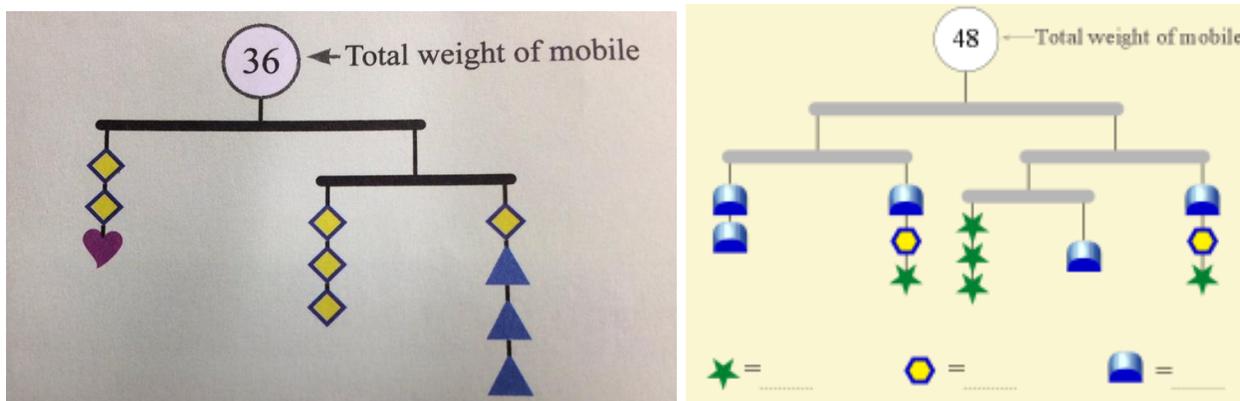
**Notes:**

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## Mobile Algebra Puzzles

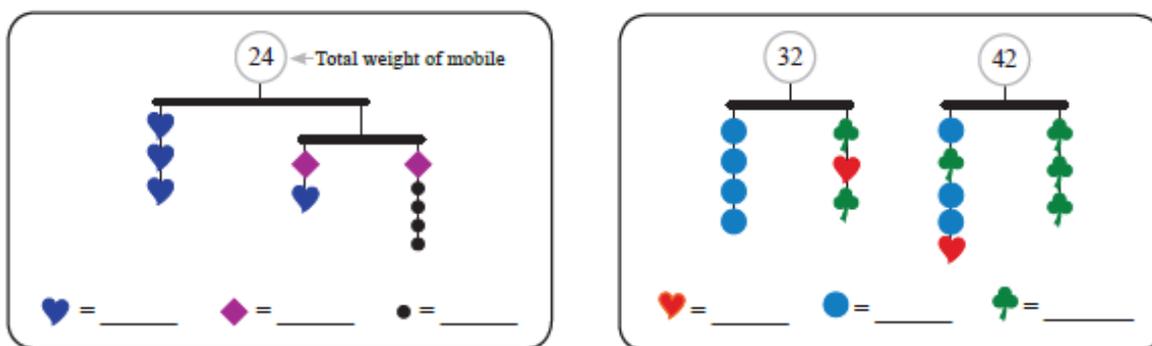
\*Credit to <http://thinkmath.edc.org/resource/mobile-puzzles> and <http://ttalgebra.edc.org/puzzles>

### Problem:



What weights do the different symbols represent if the beams are all balanced?

### Extension:



Can you make your own mobile algebra puzzle?

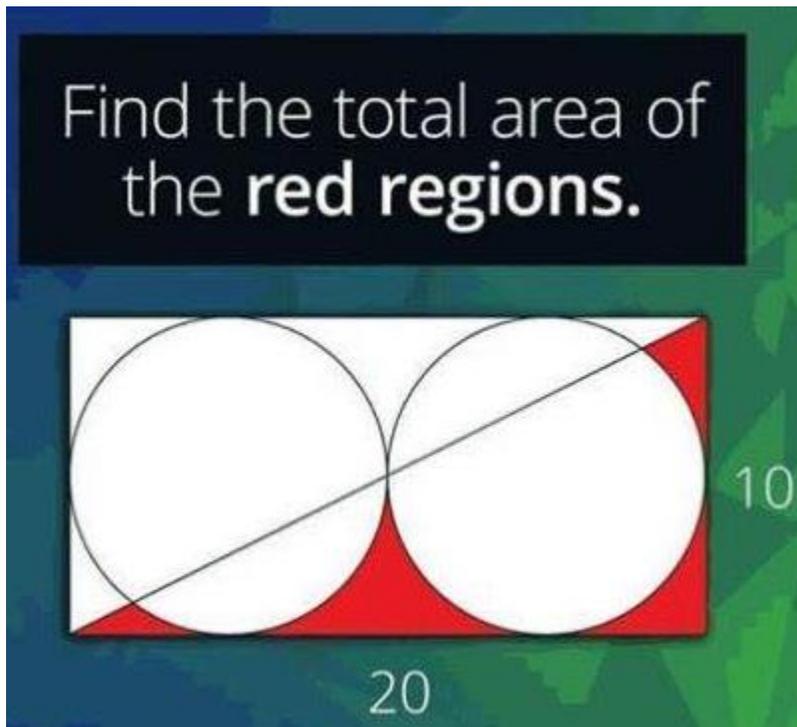
Notes: Visit <http://ttalgebra.edc.org/puzzles> for more.

[BACK](#)

## Red

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Problem:



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Extension:

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Notes:

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## Pigs and Chickens

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**Problem:**

Farmer Fiona has pigs and chickens. Last Tuesday, she counted 34 eyes and 46 feet. How many chickens does she have?

**Problem:**

Gina and Tom raise cats and birds. They counted all the heads and got 10. They counted all the feet and got 34. How many birds and cats do they have?

**Problem:**

Fred and Patti go to Grandpa's farm. In the barnyard there are ducks and sheep. Fred and Patti see 9 animals all together. The animals have a total of 26 legs. How many ducks and how many sheep are there in the barnyard?

---

**Extension:**

Make your own farm animal problem.

---

**Notes:**

# Order of Operations

*\*Credit to Robert Kaplinsky and <http://www.openmiddle.com/>*

## Problem:

Make the largest expression by using the whole numbers 0-9 once each in the boxes below.

$$\square \div \square (\square + \square)^{\square} \cdot \square - \square$$

What other expressions can you make?

## Extension:

What is the smallest expression possible?

**Notes:** Visit <http://www.openmiddle.com/> for more.

## Take a Prime

---

*\*Credit to James Tanton*

### Problem:

Take a prime, triple it and add either 1 or 2, and get another prime. Repeat.

eg  $2 \rightarrow 7 \rightarrow 23 \rightarrow 71$ .

How long of a string can you get?

---

### Extension:

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### Notes:

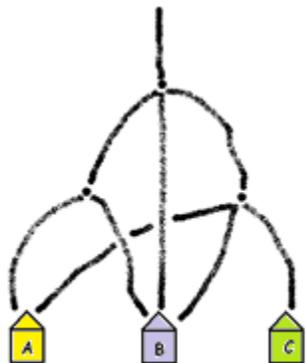
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## Down a Garden Path

*\*Credit to James Tanton*

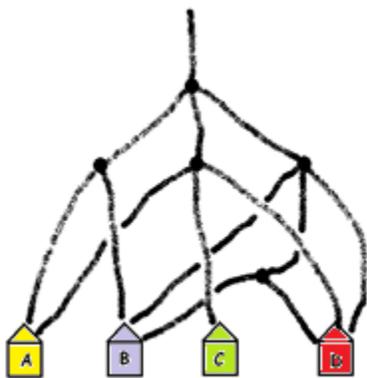
### Problem:

Folk walk down the following system of paths.



What fraction of people end up at house A, at house B, and at house C?

### Extension:



How does this relate to probability? Investigate!

**Notes:** Visit <http://gdaymath.com/lessons/powerarea/4-2-garden-paths/> to learn more.

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## Fifty-Fifty Split

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### Problem:

Erin and Aimee are each responsible for mowing half of their back yard. The yard is rectangular with the dimensions of 75m by 90m. Erin starts mowing at a corner, gradually working her way toward the middle by mowing concentric bands around the outside edges. If the mower cuts a three-foot-wide path, at what point should Erin stop and Aimee start mowing?

---

### Extension:

What if their little brother Spencer took a turn?

What if Grandma May came to help?

---

### Notes:

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## Chip in for a New Ball

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### **Problem:**

Three boys have bought a football for \$45. The first boy gave an amount that did not exceed what the remaining two boys chipped in. The second boy added no more than half of the sum paid by the first and third boy together. The third boy, however, chipped in no more than a fifth of amount contributed by the two remaining boys. How much did each boy pay for the ball?

---

### **Extension:**

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### **Notes:**

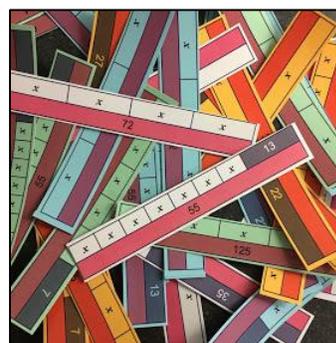
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# Equation Strips

\*Credit to <http://engaging-math.blogspot.ca/2017/01/equation-strips.html?m=1>

## Problem:

12							
$x$	$x$	$x$					
$x$	$x$	$x$	$x$	$x$			
125							
48							
$x$							
$x$	$x$	$x$	$x$				
72							
35		$x$					
52							
$x$	8						
15							
20							
11		$x$					
38							
$x$	22						



## Extension:

**Notes:** Visit blog post <http://engaging-math.blogspot.ca/2017/01/equation-strips.html?m=1>

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## KenKen Puzzle

\*Credit to <http://www.kenkenpuzzle.com/>

### Problem:

Your goal is to fill in the whole grid with numbers, making sure no number is repeated in any row or column. In a four by four puzzle, use the numbers 1 to 4. In a five by five puzzle, use the numbers 1 to 5, etc. The heavily-outlined areas are called “cages.” The top left corner of each cage has a target number and math operation. The numbers you enter into a cage must combine (in any order) to produce the target number using the math operation noted.

5+		3+
4+	3+	
		3

30,×		3, +	11, +	16,×	
	13, +			36,×	
		1, -			3
2, ÷		9,×			50,×
8, +		15, +			
			2	5, -	

**Extension:** Visit <http://thinkmath.edc.org/resource/kenken-puzzles> for more.

### Notes:

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# Mash Up Math Intro

\*Credit to <http://mashupmath.com/weekly-math-challenges/> and @mashupmath

**Problem:**

$$\text{Yellow Star} + \text{Red Mushroom} = 16$$

$$\text{Yellow Star} = \text{Fire Flower}$$

$$\text{Question Block} + \text{Question Block} = 2$$

$$\text{Fire Flower} - \text{Question Block} = 2$$

$$\text{Red Mushroom} = ?$$


**Extension:**

Many more puzzles at <http://mashupmath.com/weekly-math-challenges/>

**Notes:**

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## Mash Up Math Integers

\*Credit to <http://mashupmath.com/weekly-math-challenges/> and @mashupmath

**Problem:**

$$\text{shoe} \times \text{water bottle} + \text{bag} = \text{headphones}$$

$$\text{headphones} = 7 - 3 \times 3$$

$$\text{water bottle} + \text{shoe} = -11$$

$$\text{shoe} \times \text{water bottle} = 28$$

$$\text{headphones} \times 2 = \text{shoe}$$

$$\text{shoe} \times \text{water bottle} \times \text{headphones} + \text{bag} = ?$$

**Extension:**

Many more puzzles at <http://mashupmath.com/weekly-math-challenges/>

**Notes:**

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## Mash Up Math Decimals

\*Credit to <http://mashupmath.com/weekly-math-challenges/> and @mashupmath

**Problem:**

$$8 - \text{tent} - 1 = \text{fire}$$

$$\text{knife} = \text{fire} - 1.1$$

$$6.2 = \text{tent} + 1 + \text{tent}$$

$$\text{tent} + \text{knife} + \text{fire} = ?$$

**Extension:**

Many more puzzles at <http://mashupmath.com/weekly-math-challenges/>

**Notes:**

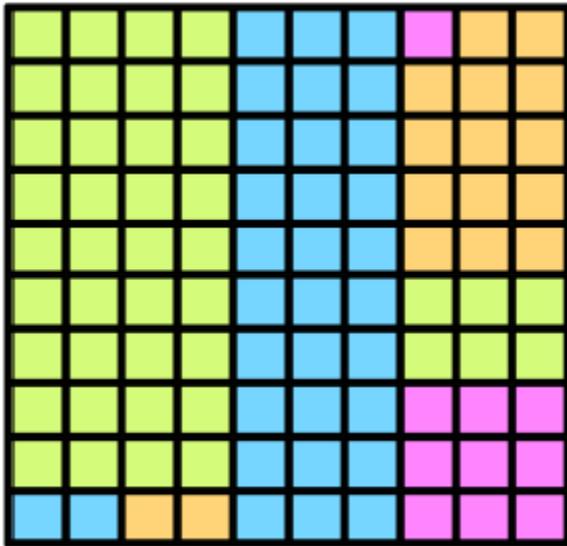
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## Mash Up Math Grid

\*Credit to <http://mashupmath.com/weekly-math-challenges/> and @mashupmath

### Problem:

If the diagram below represents 250, what is the value of each colour?



### Extension:

What if the diagram represented 1? -50? A different number?

Many more puzzles at <http://mashupmath.com/weekly-math-challenges/>

### Notes:

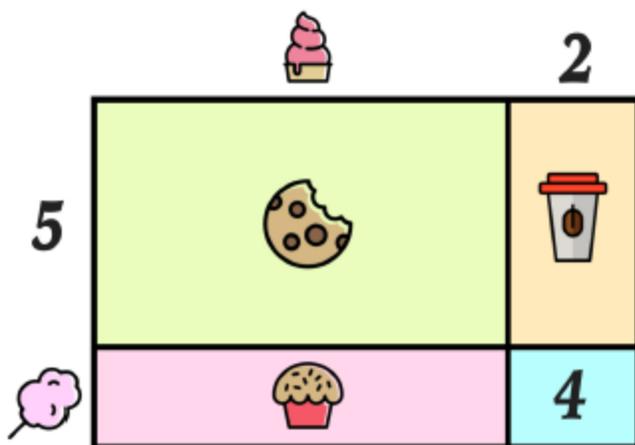
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## Mash Up Math Area Model

\*Credit to <http://mashupmath.com/weekly-math-challenges/> and @mashupmath

### Problem:

If the area model below represents the value 714, what is the value of each icon?



### Extension:

What if the diagram represented a different number? A decimal number?

Many more puzzles at <http://mashupmath.com/weekly-math-challenges/>

### Notes:

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## Mash Up Math More Integers

\*Credit to <http://mashupmath.com/weekly-math-challenges/> and @mashupmath

### Problem:

What is the value of each icon?



							
<input style="width: 40px; height: 20px;" type="text"/>							

### Extension:

What if the diagram represented a different number? A decimal number?

Many more puzzles at <http://mashupmath.com/weekly-math-challenges/>

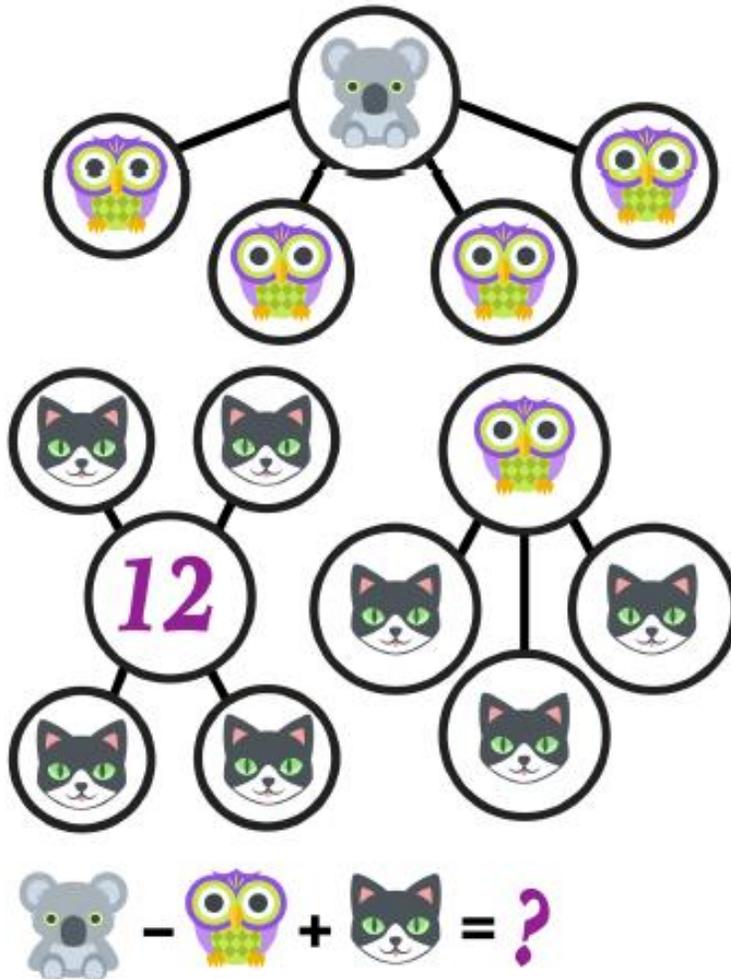
### Notes:

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## Mash Up Math Groups Of

\*Credit to <http://mashupmath.com/weekly-math-challenges/> and @mashupmath

**Problem:**



**Extension:**

Many more puzzles at <http://mashupmath.com/weekly-math-challenges/>

**Notes:**

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# Mash Up Math Fractions

\*Credit to <http://mashupmath.com/weekly-math-challenges/> and @mashupmath

**Problem:**

$\text{Glass of juice and cookie} = \begin{matrix} \text{Watermelon slice and banana} & \text{Watermelon slice and banana} \\ \text{Watermelon slice and banana} & \text{Watermelon slice and banana} \end{matrix}$

$\text{Box of snacks} = \begin{matrix} \text{Coffee cup} \times \text{Alien} & \text{Coffee cup} + \text{Coffee cup} \\ \text{Alien} \times \text{Coffee cup} \end{matrix}$

$\text{Box of snacks} + \text{Glass of juice and cookie} = 54$

$\text{Glass of juice and cookie} - \text{Box of snacks} = \text{Box of snacks}$

**Extension:**

Many more puzzles at <http://mashupmath.com/weekly-math-challenges/>

**Notes:**

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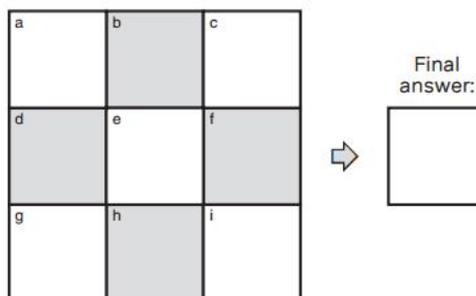
# Kakooma

*\*Credit to Greg Tang @gregtangmath*

## Problem:

Puzzles? In each 9-number square, find the number that is the sum of 2 other numbers. Use all 9 sums to create 1 final puzzle and solve.

19	16	22	14	9	15	1	12	45
46	29	21	41	2	4	20	7	35
35	39	33	31	24	25	22	5	31
30	48	3	9	17	6	2	23	8
13	31	14	49	2	50	19	36	22
4	7	47	4	12	39	29	15	50
33	20	44	49	21	35	33	22	41
30	8	34	10	13	18	26	44	29
32	45	50	4	19	40	48	31	50



## Extension:

## Notes:

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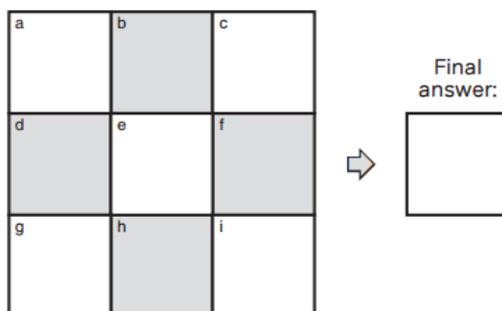
# Kakooma Integers

\*Credit to Greg Tang @gregtangmath

## Problem:

Puzzles? In each 9-number square, find the number that is the sum of 2 other numbers. Use all 9 sums to create 1 final puzzle and solve.

-4	7	5	20	21	4	18	-16	-23
-21	18	21	-25	-15	8	-24	4	-15
-6	-1	-18	-6	-5	7	0	5	-6
4	1	0	15	11	-25	-15	-21	13
16	-14	-21	14	-6	-18	18	7	1
-11	2	-6	-16	25	13	0	-9	-24
24	6	-6	-8	25	22	19	-10	-21
12	8	-17	-25	20	2	-23	-15	21
-8	-1	22	8	-9	-24	17	8	-12



## Extension:

Many more puzzles at <http://mashupmath.com/weekly-math-challenges/>

## Notes:

[BACK](#)

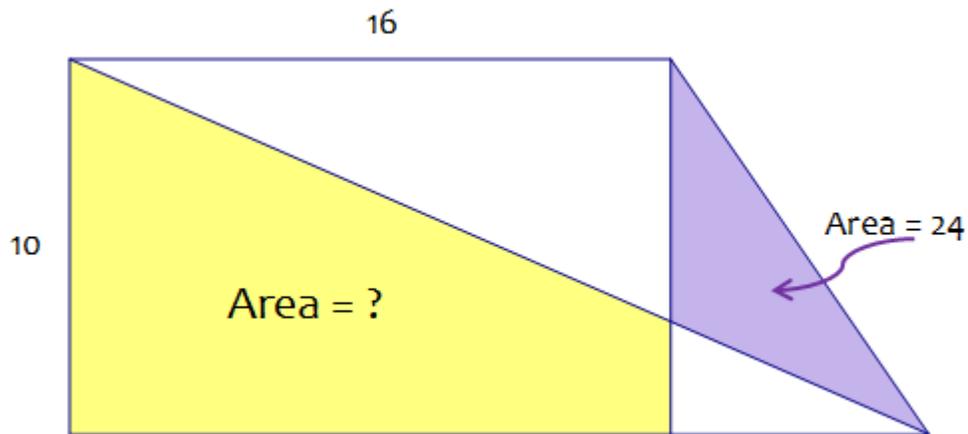
## The Missing Area

*\*Credit to Fawn Nguyen @fawnpnguyen*

---

### Problem:

A 10 by 16 rectangle is attached to a triangle as shown below. If the purple section is 24 square units, then what is the area of the yellow section of the rectangle?



---

### Extension:

---

### Notes:

Visit blog post <http://fawnnguyen.com/driving-them-nuts/>

[BACK](#)

# Twist and Rotate

---

*\*Credit to Fawn Nguyen @fawnpnguyen*

## Problem:

Start at 0, twist 4 times, rotate, twist 2 times, rotate, twist 3 times, rotate, twist 5 times. Okay, now figure out how to get back to 0.

Visit blog post for explanations <http://fawnnguyen.com/students-embroiled-conways-rational-tangles/>

---

## Extension:

What does the operation “rotate” do to the value of the ropes? We know a twist is add 1(+1), so what is rotate?

---

## Notes:

Need two long ropes of different colours per group of 4 students.

[BACK](#)

# Which is Larger: Integers

---

*\*Credit to James Tanton*

## Problem:

Which is larger for the 1<sup>st</sup>  $n$  integers: the sum of their cubes, or the square of their sum?

---

## Extension:

What do you notice? What do you wonder? Investigate!

---

## Notes:

[BACK](#)

## One Less Chocolate

---

### Problem:

Aled intends to cut down on his daily consumption of chocolates during February. Today, the first of the month, Aled keeps to his normal daily ration, but tomorrow he will eat one less, and he aims to eat one less again the following day and so on until he reaches just one chocolate a day. Once he reaches that point he intends to keep to a regime of eating just one chocolate every day. If he succeeds in his plan, Aled has worked out that he will eat a total of 56 chocolates during February. How many does Aled intend to eat today?



---

### Extension:

---

### Notes:

# Fishing

---

*\*Credit to Fawn Nguyen @fawnpnguyen*

## Problem:

One day, three people (A, B, and C) decided to go fishing. They agreed that everyone would take home one third of the total number of fish caught. After fishing, they went to sleep. At 3:00a.m., A woke up and wanted to take  $\frac{1}{3}$  of the fish and go home, but realized that the number of fish wasn't divisible by 3. So A threw one fish into the river and took  $\frac{1}{3}$ . Then at 4:00a.m., B work up thinking that A and C were still asleep. B did the same thing as A and left. Then at 5:00a.m., C was awakened by the alarm clock. C did the same thing as B and left.

What do you want to figure out?

---

## Extension:

Investigate!

What if there were more fish?

What if there were more people?

---

## Notes:

[BACK](#)

## A Measure of Sugar

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### **Problem:**

With a double pan scale and only four weights of 1-oz, 3-oz, 9-oz, and 27-oz, how does one measure 15-oz of sugar?

---

### **Extension:**

What about 25 oz of sugar?

What other amounts can you figure out?

---

### **Notes:**

[BACK](#)

## Remember Your Pin

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### Problem:

To remember certain codes or passwords, such as the PIN number, it is advisable to establish relationships between the digits that make them up since it has been noticed that such relationships tend to be retained in our memory much longer than the numbers themselves. Bill noticed that in his four-digit cell phone PIN, the second digit is the sum of the last two digits, and the first is the quotient of the last two. Moreover, the first two digits and the last two are made up of two two-digit numbers whose sum equals 100. Find Bill's cellular phone PIN.

---

### Extension:

---

### Notes:

[BACK](#)

# One Hundred-Headed Dragon

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

## Problem:

Once upon a time, there lived a fierce dragon, which had one hundred heads. With a stroke of his sword, a knight could cut off one, seven or 11 heads, but if at least one head remained uncut, immediately after the sword stroke, four, one, or five heads grew back, respectively. Was the knight able to kill the dragon?

Remember: The dragon dies if, after the sword stroke, he has no more heads.

---

## Extension:

What if the dragon had 99 heads? What if he had a different number of heads?

---

## Notes:

[BACK](#)

# The Power of a Weird Number

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

## Problem:

What do you notice about the powers of 376 (with a positive integer exponent)?

Is this always true?

Why or why not?

---

## Extension:

Can you find another weird number that behaves in a similar manner?

---

## Notes:

[BACK](#)

## Play on Numbers

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### **Problem:**

On the blackboard were the numbers: 1, 2, 3, ... all the way to 110. In each move, you were supposed to cross out any two numbers and replace them with their difference. After 109 moves, there remained on the blackboard but one number...

---

### **Extension:**

Is it always the same number?

---

### **Notes:**

[BACK](#)

## Zero-Sum Game

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### **Problem:**

Tom and Simon were taking turns rolling a die when they thought of a game: If a one is rolled by either player, Tom pays Simon 50 cents, but when a different number is rolled, Simon pays Tom 10 cents. After 30 rolls, it turned out that they were even, and neither of them won “a penny.” How many times was a one rolled?

---

### **Extension:**

What if there were two dice? What if there were three players?

---

### **Notes:**

[BACK](#)

## Fast Creepers

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### Problem:

Two snails, Daniel and Sebastian, are racing against each other along a track divided into three sections. Each section measures exactly one meter. Daniel creeps at a constant speed, whereas Sebastian covers the first section of the racetrack at a speed twice as high as Daniel, the second section at the same speed as Daniel, and the third one at half the speed of his rival. Who is going to win, and by how many meters?

---

### Extension:

What happens if you change the problem?

---

### Notes:

## Head Start for Dave

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### **Problem:**

Andrew is a far better runner than Dave, and in a 100 meter race, he breaks the finish line tape when Dave still has 20 meters to go. Their friend Joe drew an additional line 20 meters before the actual starting line and said: "Let Dave begin at the official starting line and Andrew at the new one. If they start at the same time and run at their usual speeds, they will finish the race neck and neck."

Is Joe right?

---

### **Extension:**

If not, what distance from the starting line should the new line be drawn in order that both runners reach the finishing line simultaneously?

---

### **Notes:**

## Bunnies for Sale

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### Problem:

A rabbit keeper brought his rabbits to the market. His first customer bought  $\frac{1}{6}$  of all the rabbits plus 1; the second buyer took  $\frac{1}{6}$  of the remaining rabbits plus 2; the third customer bought  $\frac{1}{6}$  of the remaining rabbits plus 3, and so on. When the man had sold all of his rabbits, he found to his surprise that each customer had bought the same number of rabbits. How many rabbits did the salesman bring to the market, and how many customers did he have?

---

### Extension:

---

### Notes:

## Roadside Villages

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### Problem:

Alongside a road, there are five villages. Let's call them A, B, C, D, and E, for short. The distance from A to D is known to be 6 km, from A to E – 16 km, from D to E – 22km, from D to C – 6 km, and from A to B – 16 km. The distances were measured along the road. Find the right order in which the villages are located along the road.

---

### Extension:

---

### Notes:

[BACK](#)

## Rectangle of Squares

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### Problem:

The rectangle presented in the figure below is made up of six squares, the smallest of them measuring 2cm by 2cm. Can you calculate the area of the rectangle? Note: The figure is not to scale!



---

### Extension:

---

**Notes:** \*Teacher Only\* hint – let “ $a$ ” be the side length of the bottom left square!

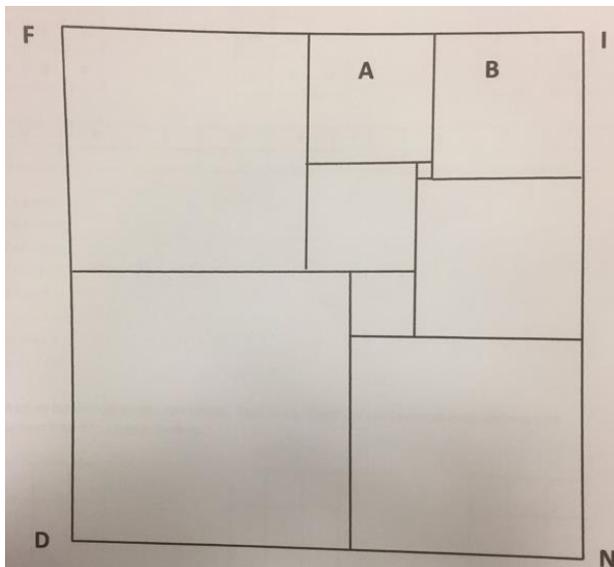
## Dissecting Squares

---

### Problem:

The area of square A is 64 square units. The area of square B is 81 square units. Can you calculate the dimensions of FIND?

Note: The figure is not to scale!



### Extension:

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### Notes:

[BACK](#)

## The Cutting Straight Line

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### Problem:

A straight line cut a square in such a way that it divided the square's perimeter in a ratio of 9:7, and two sides of the square in a ratio 7:1 and 5:3. In what ratio did the straight line divide the square's area?

---

### Extension:

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### Notes:

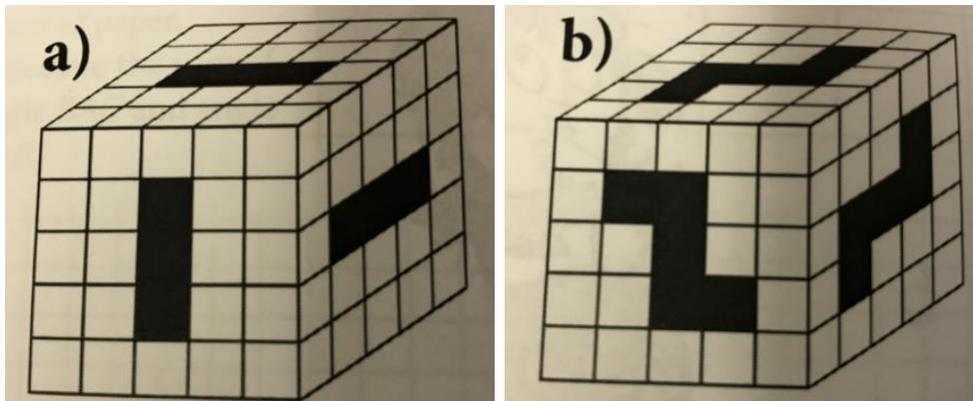
[BACK](#)

## A Cube with Holes

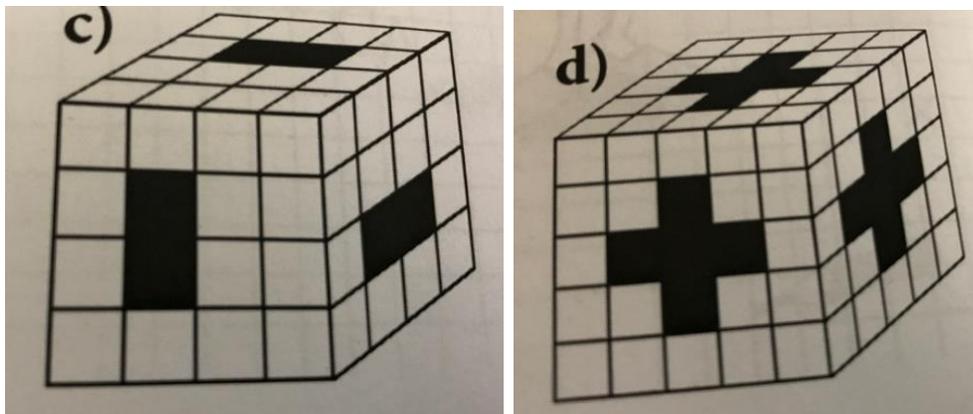
*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### Problem:

Several small cubes were glued together to form a 5 by 5 by 5 hexahedron in such a way that three hollow tunnels were created running across the whole solid. Their cross-sections were blackened in the figures below. How many small cubes were used to build each of these hexahedrons with holes in them?



### Extension:



### Notes:

[BACK](#)

## Colour Balls

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### Problem:

In a box, there are 30 one-colour balls of three different colours. If we randomly take 25 balls out of the box, among our picks will always be at least three white, at least five blue, and at least seven black balls. How many balls of each colour are there in the box?



---

### Extension:

What if there were more balls? More colours?

---

### Notes:

[BACK](#)

## Adding up to 100

---

*\*Credit to 100 Math Brainteasers by Romanowicz and Dyda*

### **Problem:**

Adam and Bill decided to have a game of adding up to 100. It is Adam who begins. His first step is to write down a natural number no greater than 10; then it is Bill's turn, who increases the number by no more than 10, but by no less than 1. Likewise, Adam increases the newly formed number by no more than 10, but by at least 1. The two players make such alternate moves until the player who first reaches 100 is pronounced the winner. Does the beginning player have a winning strategy? If so, what first move should he make, and what will be his responses to the numbers written down by his opponent?

---

### **Extension:**

What if there were more players?

---

### **Notes:**

# SKUNK

\*Credit to <http://illuminations.nctm.org/lessons/6-8/choice/worksheet.pdf>

## Problem:

Each letter of "skunk" represents a different round of the game; play begins with the "S" column and continues through the "K" column. The object of "skunk" is to accumulate the greatest possible point total over the five rounds. The rules for play are the same for each of the five rounds. To accumulate points in a given round, a pair of dice is rolled. A player gets the total of the dice and records it in his or her column, unless a "one" comes up. If a "one" comes up, play is over for that round and all the player's points in that column are wiped out. If "double ones" come up, all points accumulated in prior columns are wiped out as well. If a "one" doesn't occur, the player may choose either to try for more points on the next roll or to stop and keep what he or she has accumulated.

**Note:** If a "one" or "double ones" occur on the very first roll of a round, then that round is over and the player must take the consequences.

## Game of SKUNK



S	K	U	N	K



## Extension:

Probability, other games or experiments.

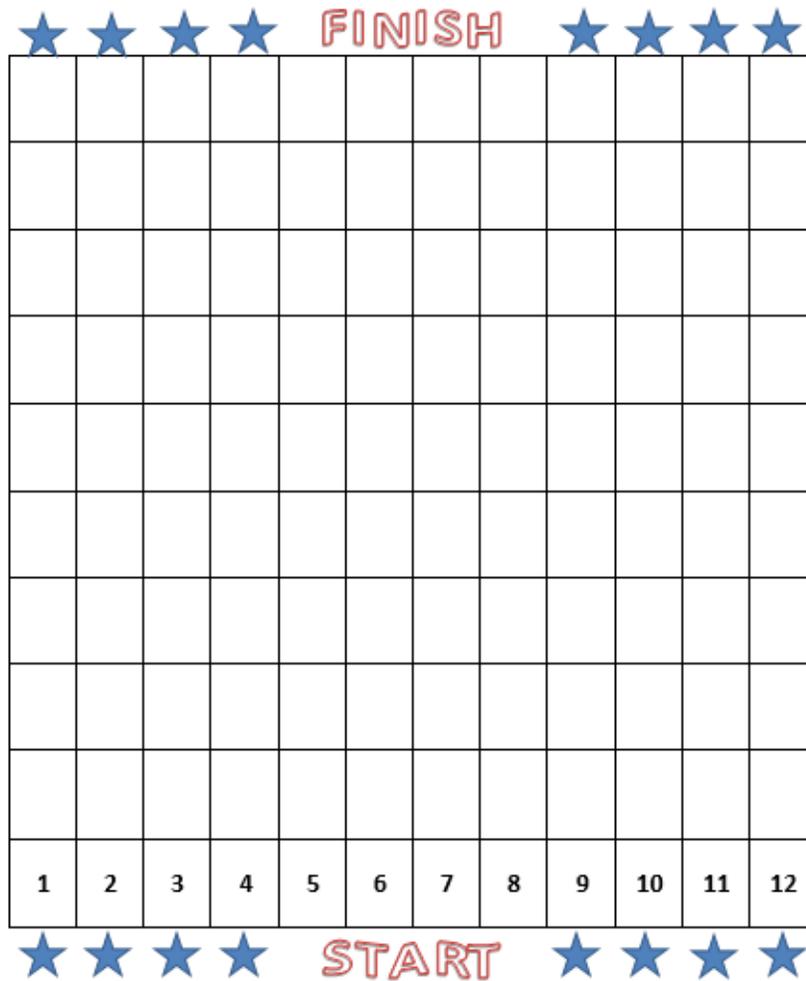
**Notes:** This is fun to do as a whole class. Stand up if you're in. Sit down if you're out.

[BACK](#)

# Horse Races

\*Credit to <https://www.tes.com/teaching-resource/probability-horse-racing-game-6338302>

## Problem:



Students choose a horse (or two or three) to bet on, numbered between 1 and 12. Two dice are then rolled and the horse with that total moves forward.

## Extension:

How did you pick your horse? What horse would you pick if you played the game again?

**Notes:** Probability

[BACK](#)

# Wine Chests

---

*\*Credit to Peter Liljedahl*

## Problem:

I like wine, red wine in particular. My wife and I drink a bottle of red wine a day. But I'm a bit of a wine snob. I've done a lot of research on wine and I've learned about the things that affect wine – the biggest thing that affects the quality of wine is light – that's why they put it in green bottles. So, I won't drink wine that has been exposed to light more than 10 times. I live out of the city, and I don't want to buy a bottle of wine every day, so I need to buy in bulk. To help, I have installed a wine chest in my house. The problem is that every time I open the door, I expose all of the wine bottles. To compensate for this problem, I have a smaller chest right next to it. I take wine out of the big chest, move it to the smaller chest, and then take it out to drink it.

How often do I have to buy wine?

---

## Extension:

---

## Notes:

I buy it in boxes so it's not exposes prior to me getting it. I open all of the boxes to store the wine all at the same time.

Opening a chest exposes all of the bottles in that chest.

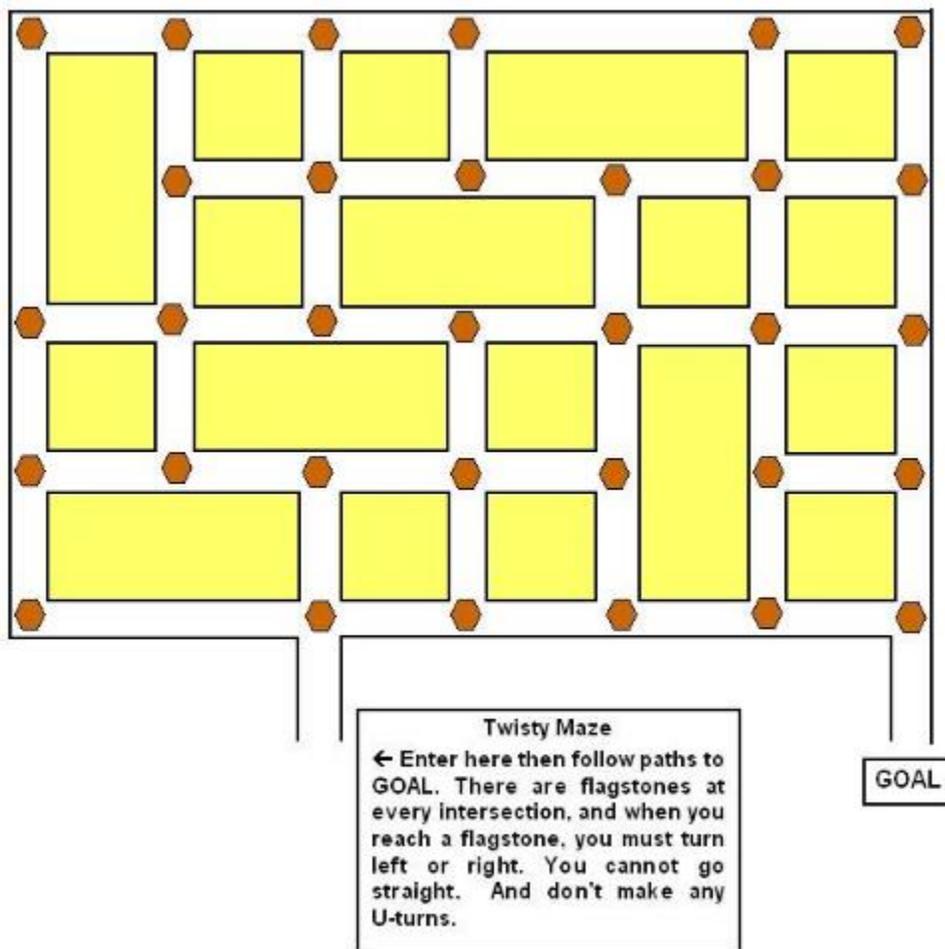
Maybe pop or juice instead of wine for children?

# Twisty Maze

*\*Credit to Robert Abbott and <http://www.logicmazes.com/>*

## Problem:

The diagram below shows a Twisty Maze, along with the sign that gives its rules. This is a small walk-through maze that the creator uses outside of some of his large cornfield mazes.



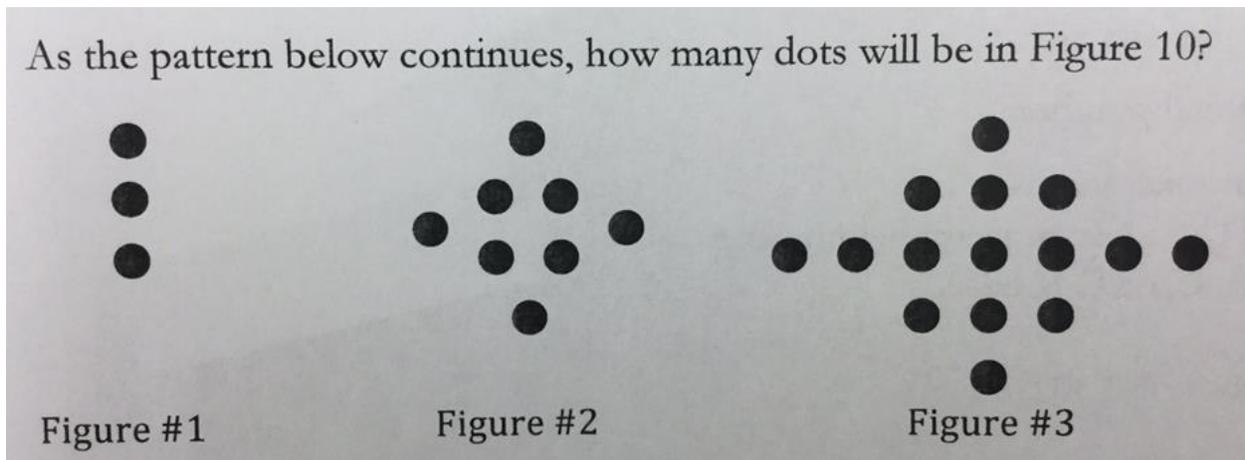
**Notes:** Visit <http://www.logicmazes.com/>

[BACK](#)

## Figure 10

---

### Problem:



---

### Extension:

Make your own pattern, figure out the expression, and then share it with another group.

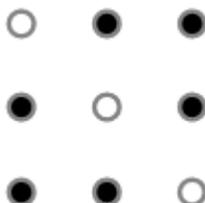
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### Notes:

## Just Two Dots

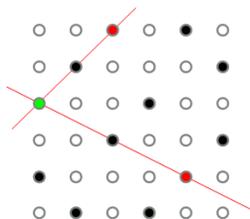
\*Credit to <http://galileo.org/classroom-examples/math/math-fair-problems/puzzles/minaret-with-a-view/>

### Problem:



Minarets in the utopian downtown of New Istanbul will be so beautiful that it will be the law that from every minaret, every other minaret may be seen. When the downtown was small this was relatively easy. The above illustration shows 6 minarets in a downtown of  $3 \times 3$  blocks.

Below, in a  $6 \times 6$  downtown, a mistake has been made because a person on top of the green minaret cannot see either of the red minarets:



There are two solutions for placing 8 minarets when the downtown grows to  $4 \times 4$ . Can you find both of them?

### Extension:

How many minarets can you place in a  $5 \times 5$  utopian downtown?

What is the maximum number of minarets you can place in an  $N \times N$  utopian downtown?

Is there any limit to the size of a utopian downtown if its  $N \times N$  blocks must contain  $2N$  minarets? **Warning:** This is an unsolved problem in mathematics

**Notes:** <http://galileo.org/classroom-examples/math/math-fair-problems/puzzles/minaret-with-a-view/>

[BACK](#)

## Road Trip

---

### Problem:

Betty and Tracy planned a 5000km trip in an automobile with five tires, of which four are in use at any time. They plan to interchange them so that each tire is used the same number of kilometers. What is the number of kilometers each tire will be used?

---

### Extension:

What if the trip was longer? What if they had more tires?

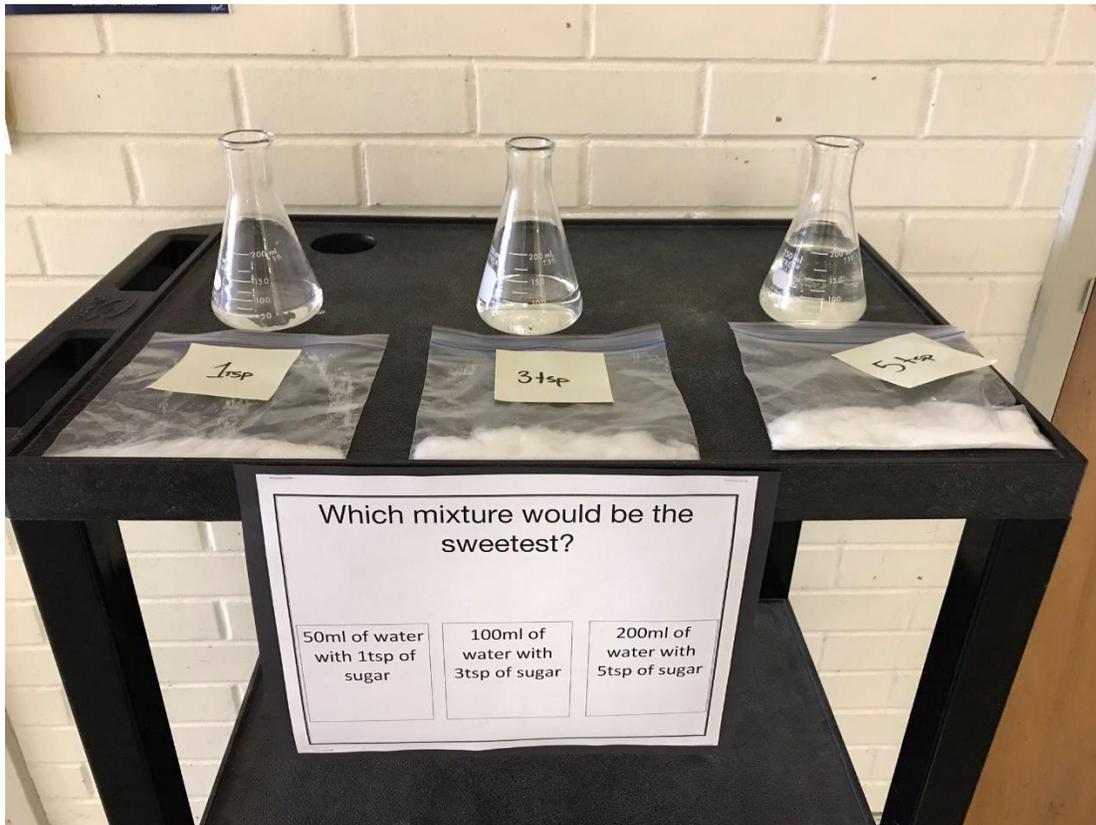
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### Notes:

# Sweetest

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**Problem:**



---

**Extension:**

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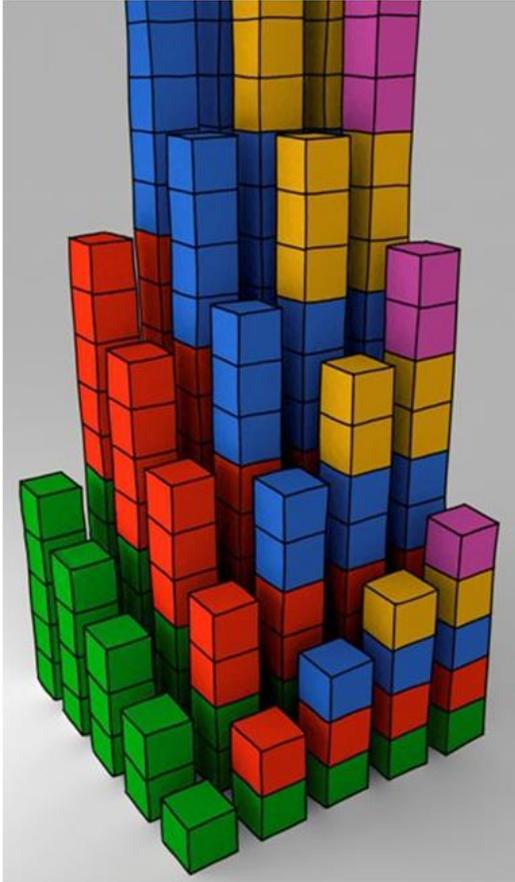
**Notes:**

[BACK](#)

# Math Art

---

**Problem:**



What do you notice? What do you wonder?

---

**Extension:**

What do you want to create?

---

**Notes:**

[BACK](#)

## Black and White Marbles

---

*\*Credit to Vector 55(1) - p. 49*

### Problem:

We place in a box thirteen white marbles and fifteen black marbles. We also have twenty-eight black marbles outside the box. We randomly remove two marbles from the box. If they have a different colour, we put a white one back in the box. If they have the same colour, we put a black marble in the box. We continue doing this until only one marble is left in the box. What is its colour?

---

### Extension:

Is this always true? Why or why not?

What if the numbers of marbles were different?

---

### Notes:

[BACK](#)

# Lamp Posts

---

*\*Credit to Vector 55(1) - p. 49*

## Problem:

In a new development there is a lamp post placed at every intersection of roads. What is the greatest number of lamp posts that will be required for a given number of streets?

---

## Extension:

How do you know this is the greatest number?

---

## Notes:

[BACK](#)

# Investigate Polyhedrons and Platonic Solids

---

*\*Credit to Peter Liljedahl*

## Problem:

Build and investigate 3 dimensional objects with bendy straws. Cut a slit in each of the bendy parts and connect with the other end of another straw.

---

## Extension:

Shapes of faces, Vertex, Euler's Formula, Vertex Peak, Sum of angles...

---

## Notes:

Bendy straws and tape

[BACK](#)

# Diagonals in a Rectangle

---

*\*Credit to Rina Zazkis*

## Problem:

Given a  $n \times k$  rectangle drawn on graph paper, how many grid squares are crossed by its diagonal?

---

## Extension:

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## Notes:

[BACK](#)

## Chocolate, Chocolate, Chocolate

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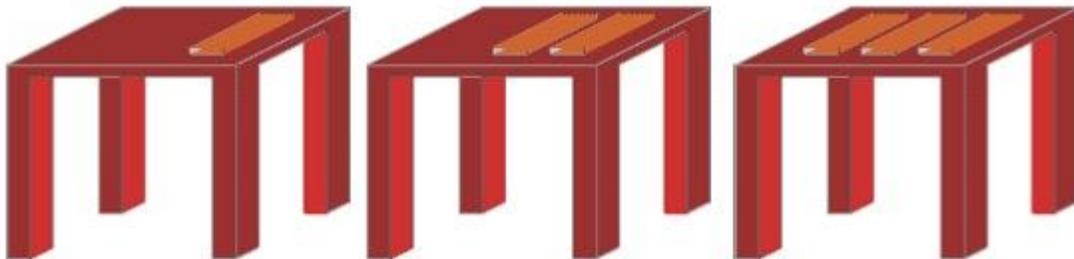
\*Credit to <http://nrich.maths.org/public/leg.php?code=27&cl=2&cldcmpid=34>

### Problem:

There's a room in your school that has three tables in it; table 1 has one block of chocolate on it, table 2 has two blocks of chocolate on it and, table 3 has three blocks of chocolate on it.

Now ... outside the room is a class of children. Thirty of them all lined up ready to go in and eat the chocolate. These children are allowed to come in one at a time and can enter when the person in front of them has sat down. When a child enters the room they ask themselves this question:

***"If the chocolate on the table I sit at is to be shared out equally when I sit down, which would be the best table to sit at?"***



The chocolate is not shared out until all the children are in the room!

---

### Extension:

---

**Notes:** Visit <http://nrich.maths.org/public/leg.php?code=27&cl=2&cldcmpid=34>

[BACK](#)

# Snake Pattern

---

*\*Credit to Rina Zazkis*

## Problem:

```
  1  2  3  4
    8  7  6  5
  9 10 11 12
    16 15 14 13
 17 18 19 20
   ...           21
```

How can you continue this pattern?

Suppose you continue it indefinitely. Are there numbers that you know "for sure" where they will be placed? How did you decide?

---

## Extension:

Can you predict where the number 50 will be? 150? And how about 86? 87?  
187? 392? 7386? 546?

In general, given any whole number, how can one predict where it will appear in this pattern? Explain the strategy that you propose.

---

## Notes:

[BACK](#)

## Triangular Numbers and Square Numbers

---

### Problem:

There are several interesting connections between these 2 sets of numbers, and also among the numbers in each set. Your task is to identify several connections and explain why they exist.

---

### Extension:

---

### Notes:

## Counting Factors

---

### Problem:

Find 3 numbers with exactly 7 factors.

Find 3 numbers with exactly 8 factors.

Find 3 numbers with exactly 20 factors.

How do you do this?

Is it efficient?

---

### Extension:

How can you generate a “large” number with exactly 7 factors.

How can you generate a “large” number with exactly 8 factors.

---

### Notes:

[BACK](#)

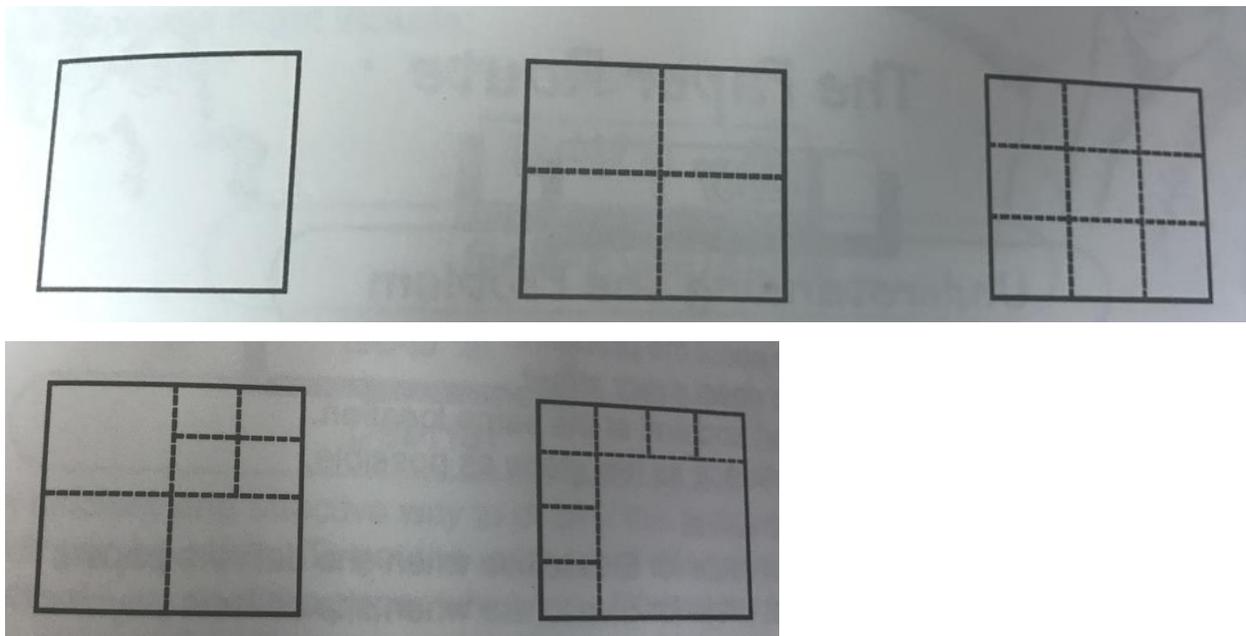
# Cutting Cardboard Squares

---

*\*Credit to Rina Zazkis*

## Problem:

How many different ways can I cut a square piece of cardboard into squares?  
There can't be any extra cardboard leftover. Here are some examples.



---

## Extension:

How can I come up with different amounts of squares?

What numbers can't be represented?

Can I find any patterns?

---

## Notes:

[BACK](#)

# Divisibility Rules

---

*\*Credit to Rina Zazkis*

## Problem:

Can you come up with a rule to determine if a number is divisible by 1, by 2, by 3, by 4, by 5, by 6, by 7, by 8, by 9, by 10?

Start with the easy ones!

---

## Extension:

---

## Notes:

[BACK](#)

# Barcodes

---

*\*Credit to Rina Zazkis*

## Problem:

Universal Product Codes (UPCs) are used to identify retail products. The codes have 12 digits, and sometimes start with 0. To check that a UPC is valid, follow these steps:

Add the digits in the odd-numbered positions

Multiply this sum by 3.

To this product, add the digits in the even-numbered positions.

The result should be a number divisible by 10.

Find some products and check their UPCs. Is this always true?

---

## Extension:

---

## Notes:

[BACK](#)

# ISBN Codes

---

*\*Credit to Rina Zazkis*

## Problem:

Investigate the ISBN code. It is a 10-digit code where the last digit is the “check” digit. If the first 9 digits are  $d_1, d_2, d_3, \dots, d_9$ , the following operation is applied:

$$10 \times d_1 + 9 \times d_2 + 8 \times d_3 + 7 \times d_4 + 6 \times d_5 + 5 \times d_6 + 4 \times d_7 + 3 \times d_8 + 2 \times d_9 =$$

To the result the last digit  $d_{10}$  is added such that the total is divisible by 11.

More information is available for example on <http://en.wikipedia.org/wiki/ISBN>

If one of the first 9 digits in the code is missing, how can this digit be determined?

---

## Extension:

Find out how some of the other codes are verified (passports, airplane tickets, bank accounts, social insurance numbers etc.)

---

## Notes:

[BACK](#)

## Mancala – the Oldest Game in the World

---

<b>MANCALA</b>							<b>MANCALA</b>

1. Place 4 beans or rocks in each of the smaller cells.
2. A player moves by picking up all the blocks from one of their cells and seeding them to their right/counter clockwise (placing one block in each cell including the CALA).
3. If the last block is placed in your own CALA then you get to go again.
4. The first player to clear their side wins.

---

**Extension:** What is your winning strategy? Why does it work?

---

**Notes:** There are Mancala game boards in stores, online games, or you can make them out of egg cartons.

[BACK](#)

## Power of Numerical Variation

---

*\*Credit to Rina Zaskis*

### **Problem:**

I want to buy one kg of grains – the cost is \$0.54

I only have \$0.45. How much grain can I buy for \$0.45?

(If the grain is \$2 and I have \$10, how much grain can I buy?)

### **Problem:**

A can of coffee costs \$10. There is a shortage of coffee beans, so the price increases by 400%

(Try 20% increase - 20% of 10 is two, so \$12)

### **Problem:**

A bottle of wine costs \$5 – I pay 120% tax!!! How much do I pay?

(How could you change the numbers to better understand the problem?)

---

### **Extension:**

---

**Notes:** Change the numbers to understand the problem first.

[BACK](#)

# Rumour Mill

---

## Problem:

A rumour started at school with one student telling two others that there was to be a new holiday on Wayne Gretzky's birthday, January 26<sup>th</sup>. The two students were told that the next day they must each repeat the rumour to two more students. Each of these new students was to repeat the rumour the next day, to two more students. How many students would hear the rumour each day? How fast would it take the rumour to spread throughout your whole school? How long would it take the rumour to spread through your city, your province, your country, the world?

---

## Extension:

How long would it take the rumour to spread through your city, your province, your country, the world?

What if 3 students were spreading the rumour every day?

---

## Notes:

## More Painted Cubes

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### Problem:

The letters A to M are being painted on wooden cubes. A is painted on one cube, B is painted on two cubes, C is painted on three cubes and so on. How many blocks are painted altogether?

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### Extension:

What if we went all the way to Z?

---

### Notes:

# Dueling Dice

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\*Credit to [Mathematics Task Centre](#)

## Problem:

Consider the following four dice and the numbers on their faces:

- Red : 0, 1, 7, 8, 8, 9
- Blue: 5, 5, 6, 6, 7, 7
- Green: 1, 2, 3, 9, 10, 11
- Black: 3, 4, 4, 5, 11, 12

These are used to play a game for two people. Player 1 chooses one of the die to use for the game. Then player 2 chooses a die. Now each player rolls their die. The player with the highest number showing gets a point. The first player to 7 points wins the game. If you are player 1 which die should you choose? If you are player 2 which die should you choose?

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## Extension:

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## Notes:

[BACK](#)

# Sum of 51

---

\*Credit to [Practice Fermat Number 4, #7](#)

## Problem:

How many 6 digit numbers are there whose digits sum to 51?

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## Extension:

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## Notes:

[BACK](#)

# 105

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\*Credit to [Canadian Team Math Contest, 2014, #12](#)

## Problem:

In how many ways can 105 be expressed as the sum of at least two consecutive positive integers?

---

## Extension:

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## Notes:

# Tax Man

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\*Credit to [Numberplay](#), April 13

## Problem:

Tax Man is played like this: Start with a collection of paychecks, from \$1 to \$12. You can choose any paycheck to keep. Once you choose, the tax collector gets all paychecks remaining that are factors of the number you chose. The tax collector must receive payment after every move. If you have no moves that give the tax collector a paycheck, then the game is over and the tax collector gets all the remaining paychecks. The goal is to beat the tax collector.

## Example:

Turn 1: Take \$8. The tax collector gets \$1, \$2 and \$4.

Turn 2: Take \$12. The tax collector gets \$3 and \$6 (the other factors have already been taken).

Turn 3: Take \$10. The tax collector gets \$5.

You have no more legal moves, so the game is over, and the tax collector gets \$7, \$9 and \$11, the remaining paychecks.

## Total Scores:

You:  $\$8 + \$12 + \$10 = \$30$ .

Tax Collector:  $\$1 + \$2 + \$3 + \$4 + \$5 + \$6 + \$7 + \$9 + \$11 = \$48$ .

## Questions:

Is it possible to beat the tax collector in this \$12 game? If so, how? What is the maximum score you can get?

---

## Extension:

What if you played the game with paychecks from \$1 to \$24? How about \$1 to \$48?

---

## Notes:

# Cereal Prizes

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*\*Credit to [George Reese](#)*

## Problem:

Suppose there was one of six prizes inside your favorite box of cereal. Perhaps it's a pen, a plastic movie character, or a picture card. How many boxes of cereal would you expect to have to buy, to get all six prizes?

---

## Extension:

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## Notes:

[BACK](#)

# Tower of Hanoi

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## Problem:



The object of this puzzle is to move the tower of rings (tower of rings from one peg to another following these two rules:

You can only move one ring at a time.

You can never place a ring on top of a smaller ring.

Here is an online simulation:

<https://www.mathsisfun.com/games/towerofhanoi.html>

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## Extension:

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## Notes:

[BACK](#)

# Age Old Problems

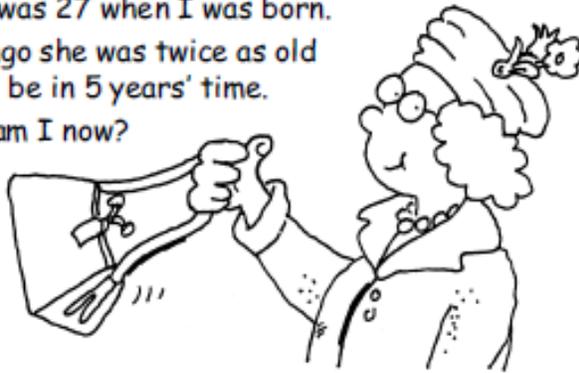
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*\*Credit to Mathematical Challenges for Able Pupils*

**Problem:**

## Age old problems

1. My age this year is a multiple of 8.  
Next year it will be a multiple of 7.  
How old am I?
2. Last year my age was a square number.  
Next year it will be a cube number.  
How old am I?  
How long must I wait until my age is both  
a square number and a cube?
3. My Mum was 27 when I was born.  
8 years ago she was twice as old  
as I shall be in 5 years' time.  
How old am I now?



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**Extension:**

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**Notes:**

# Four by Four

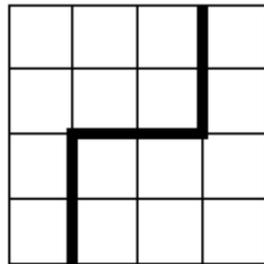
*\*Credit to Mathematical Challenges for Able Pupils*

## Problem:

### Four by four

You need some squared paper.

This 4 by 4 grid is divided into two identical parts. Each part has the same area and the same shape.



Find five more ways of dividing the grid into two identical parts by drawing along the lines of the grid. Rotations and reflections do not count as different!

Explore ways of dividing a 4 by 4 grid into two parts with equal areas but different shapes.

## Extension:

Could you find three identical parts?  
What if it was a 5 by 5 grid?

## Notes:

# Happy Numbers

---

*\*Credit to Mark Chubb and <https://buildingmathematicians.wordpress.com/2016/06/11/happy-numbers/amp/>*

## Problem:

A **happy number** is a number defined by the following process: Starting with any positive integer, replace the number by the sum of the squares of its digits, and repeat the process until the number either equals 1 (where it will stay), or it loops endlessly in a cycle which does not include 1 (definition from Wikipedia).

Numbers that result in the number 1, remain at 1 and are therefore HAPPY.

Numbers that start to loop will not result in the number 1, and are therefore called SAD numbers.

How many happy numbers can you find?

---

## Extension:

Can you predict which numbers will be happy?

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## Notes:

[BACK](#)

## Mathcounts Challenge

\*Credit to [https://mobile.nytimes.com/2017/05/15/us/math-counts-national-competition.html?\\_r=0&referrer=https://t.co/N9L2BLNC8k](https://mobile.nytimes.com/2017/05/15/us/math-counts-national-competition.html?_r=0&referrer=https://t.co/N9L2BLNC8k)

### Problem:

1. In a barn, 100 chicks sit peacefully in a circle. Suddenly, each chick randomly pecks the chick immediately to its left or right. What is the expected number of **un-pecked** chicks?
2. The smallest integer of a set of consecutive integers is -32. If the sum of these integers is 67, how many integers are in the set?
3. A bag of coins contains only pennies, nickels and dimes with at least five of each. How many different combined values are possible if five coins are selected at random?

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### Extension:

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### Notes:

[BACK](#)

## Brilliant.org Challenge

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\*Credit to <https://www.google.ca/amp/s/amp.businessinsider.com/if-you-can-solve-these-math-problems-you-are-as-smart-as-the-worlds-smartest-teenagers-2013-5>

### Problem:

1. A grid with 3 rows and 52 columns is tiled with 78 identical  $2 \times 1$  dominoes. How many ways can this be done such that exactly two of the dominoes are vertical. Note: The dominoes will cover the entire board. They are not allowed to jut over the board, or overlap with each other. Rotations and reflections count as distinct ways.
2. While studying for a test, Calvin absentmindedly eats through 7 bags of gummy worms. He looks up the nutritional information on the bag, and notices that each bag of gummy worms contains 6 servings, and each serving has a whole number of calories. Given this, Calvin calculated that he consumed just under 1000 calories. What is the maximum possible number of calories that Calvin could have consumed from these 7 bags of gummy worms?
3. How many 4-digit numbers are there such that the thousands digit is equal to the sum of the other 3 digits?
4. Over the summer, you, along with a group of friends, went to Ten Flags Amusement Park. There are 10 “must ride” roller coasters. At the end of the day, everyone rode exactly 5 of these 10 rides (due to the really long lines). Furthermore, any 2 different people rode at most 2 rides in common. What is the highest possible number of people in this group?

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### Extension:

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### Notes:

[BACK](#)

## Math Problems No One Can Solve

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\*Credit to <http://www.popularmechanics.com/science/q2816/5-simple-math-problems/>

### Problem:

#### Inscribed Square

Draw a closed loop. The loop doesn't have to be a circle, it can be any shape you want, but the beginning and the end have to meet and the loop can't cross itself. It should be possible to draw a square inside the loop so that all four corner of the square are touching the loop. According to the inscribed square hypothesis, every closed loop (specifically every plane simple closed curve) should have an inscribed square, a square where all four corners lie somewhere on the loop. This has already been solved for a number of other shapes, such as triangles and rectangles. But squares are tricky, and so far a formal proof has eluded mathematicians.

### Problem:

#### Happy Endings

Make five dots at random places on a piece of paper. Assuming the dots aren't deliberately arrange – say, in a line – you should always be able to connect four of them to create a convex quadrilateral, which is a shape with four sides where all of the corners are less than 180 degrees. The gist of this theorem is that you'll always be able to create a convex quadrilateral with five random dots, regardless of where those dots are positioned.

How many dots would you need to always be able to create a convex pentagon? What about a hexagon?

Can you figure out a formula for any of these situations?

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### Extension:

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### Notes:

[BACK](#)

## Uncracked 33

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*\*Credit to Diophantus of Alexandria, c. 215-290*

### Problem:

Which values of  $n$  from 0-100 are possible by summing the cubes of three positive integers ( $a^3 + b^3 + c^3 = n$ )?

---

### Extension:

Can the cubes be negative?

---

### Notes:

\*Teacher Only\*: This is unsolved as you can see in the video, but do not tell your students this till the end of class! This video might be the best way to end the class – or to start the next class: <http://mathpickle.com/project/uncracked-33-negative-integers-exponents/>

[BACK](#)

## When Was a Million Seconds Ago?

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\*Credit to vlogbothers at <https://youtu.be/cJ7A0yTDiqQ>

### Problem:

When was a million seconds ago?

---

### Extension:

When was a trillion seconds ago?

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### Notes:

\*Teacher Only\*: Make sure you pause the video at 0:59 before it gives the answers.

[BACK](#)

# Mathematics, Magic, and Mystery

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*\*Credit to mathaware.org*

## Problem:

Mathematics, Magic, and Mystery <http://www.mathaware.org/mam/2014/>

- 30 days of videos and articles on mathematical magic tricks, mysteries, puzzles, illusions, and more!

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## Extension:

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## Notes:

[BACK](#)

## Two Darts

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*\*Credit to Peter Liljedahl*

### Problem:

You have two darts that you can throw over and over again. The dart board has 2 rings. One is worth 4 points if you hit it, the other is worth 9 points. How can you make 26 points? How can you make 19 points? Are there numbers you can't get?

---

### Extension:

What is the biggest number you can't get?

What if you had two rings, 3 and 7?

Two rings, 2 and 11?

Two rings, 6 and 9?

Three rings, 2, 5, and 7?

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### Notes:

# Kieron's Cats

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*\*Credit to Mathematical Challenges for Able Pupils*

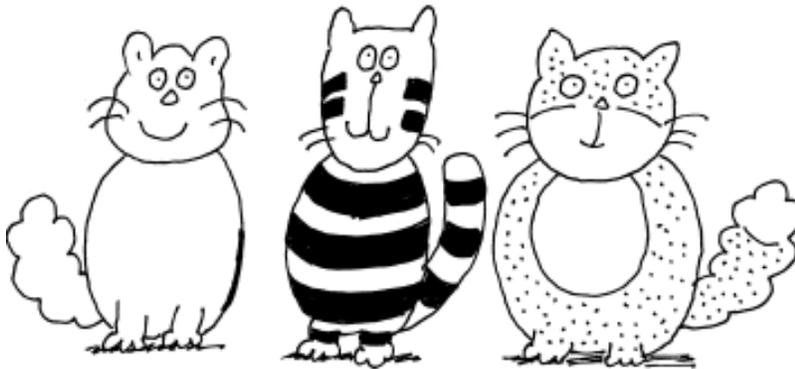
## Problem:

### Kieron's cats

Kieron has three cats.  
Each is a different weight.

The first and second weigh 7 kg altogether.  
The second and third weigh 8 kg altogether.  
The first and third weigh 11 kg altogether.

What is the weight of each cat?



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## Extension:

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## Notes:

[BACK](#)

# Candle's Burning

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*\*Credit to Kyle Pearce*

## Problem:

How long will it take for the candle to burn out?

<https://tapintoteenminds.com/3act-math/candles-burning/#task>

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## Extension:

What's the relationship between candle length and burning time?

---

## Notes:

[BACK](#)

## Consecutive Odd Numbers

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### Problem:

What happens when we add consecutive odd numbers? Can you show this with manipulatives or a picture?

Start with one, add three, then five, then seven....

---

### Extension:

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### Notes:

# Pythagorean Triples

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## Problem:

Pythagorean triples are positive integers  $(a, b, c)$  such that  $a^2 + b^2 = c^2$ . For example: 3, 4, and 5 make up a Pythagorean triple.

How many Pythagorean triples can you find?

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## Extension:

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## Notes:

For more information and a visual representation of Pythagorean triples: <https://www.youtube.com/watch?feature=youtu.be&v=QJYmyhnaaek&app=desktop>

[BACK](#)

# Drive Yourself Nuts!

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*\*Credit to John Firkins, Gonzaga University, Problem Solving in the Mathematics Classroom, MCATA*

## Problem:

Place 10 nuts in five lines of four nuts each.

---

## Extension:

Place 12 nuts in six lines of four nuts each.

Place 16 nuts in eight lines of four nuts each.

Any other numbers you want to try?

How about 10 nuts in 45 lines of two nuts each?

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## Notes:

[BACK](#)

# The Great Rope Robbery

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*\*Credit to Elliott Bird, Long Island University, Problem Solving in the Mathematics Classroom, MCATA*

## Problem:

Two ropes hang 30 centimetres apart in a tall room, 10 metres from floor to ceiling. A rope thief with a sharp knife wants to take as much rope as possible, but while the thief can climb as high as necessary, a jump of more than 330 centimetres results in death. How much rope can the thief steal?

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## Extension:

Hints: Here are some answers to provide as the questions come up. How are the ropes attached to the ceiling? With very strong nails. What if there was a hook? There isn't!

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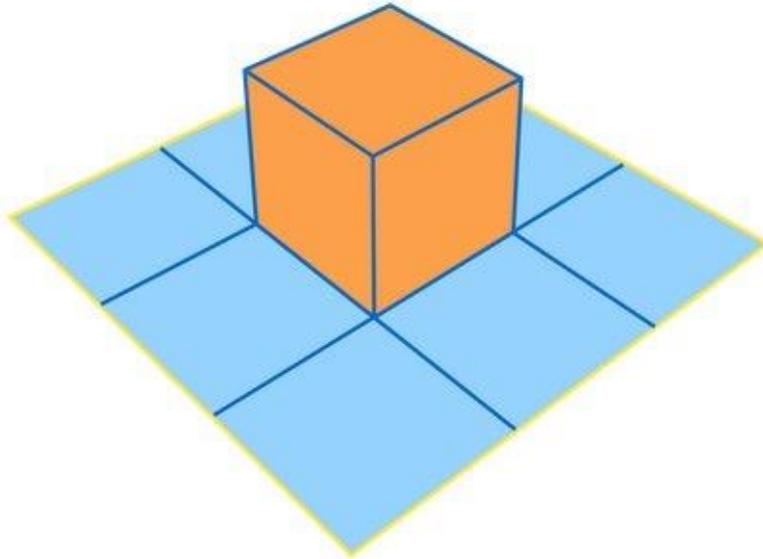
## Notes:

[BACK](#)

## Orange Cube

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### Problem:



We have a  $1 \times 1 \times 1$  orange cube on a piece of  $3 \times 3$  blue paper. **Is it possible to cover the entire cube with the paper under the following conditions?**

1. The paper can only be cut or folded along the grid.
2. The cut should not cause the paper to separate into pieces.

---

### Extension:

What if it was a larger cube? What if it was a larger piece of paper?

---

### Notes:

## Nadia's Theory

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*\*Credit to Ma, 1999, p.84*

### Problem:

Nadia comes into class very excited. She tells you (the teacher) that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:

The diagram shows two squares. The first square has a side length of 4cm. Below it, the text reads: Perimeter = 16 cm, Area = 16 square cm. The second square has a side length of 8cm. Below it, the text reads: Perimeter = 24 cm, Area = 32 square cm.

What do you think of Nadia's Theory?

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### Extension:

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### Notes:

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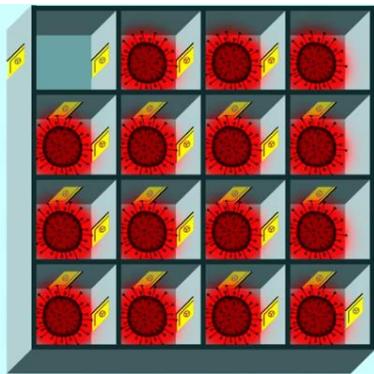
## Can You Solve the Virus Riddle?

*\*Credit to Lisa Winer – ed.ted.com*

### Problem:

Your research team has found a prehistoric virus preserved in the permafrost and has isolated it for study. After a late night working, you're just closing up the lab when a sudden earthquake hits and breaks all of the sample vials.

The virus is contained, for now. Unless you can destroy it, the vents will soon open and unleash a deadly airborne plague! Without hesitation, you grab your hazmat suit and get ready to save the world. The lab is a 4 by 4 compound of 16 rooms, with an entrance at the north-west corner, and an exit at the south-east corner. Each room is connected to the adjacent rooms by an airlock. The virus has been released in every room, except for the entrance. To destroy it, you must enter each contaminated room and pull its emergency self-destruct switch. But, there is a catch... Because the security system is on lockdown, once you enter a room, you can't exit without activating the self-destruct switch. Once, you've done so, you won't be able to go back into that room. How can you destroy the virus in every contaminated room and live to tell the story?



1. You must enter the building through the entrance and leave through the exit.
2. Every room except the entrance is contaminated.
3. Once you enter a contaminated room, you must pull the switch.
4. After pulling the switch, you must immediately leave the room.
5. You cannot return to a room after its switch has been activated.

### Extension:

What if the complex was a different size? What if it was rectangular?

**Notes:** Here is the video:

<http://ed.ted.com/lessons/can-you-solve-the-virus-riddle-lisa-winer>

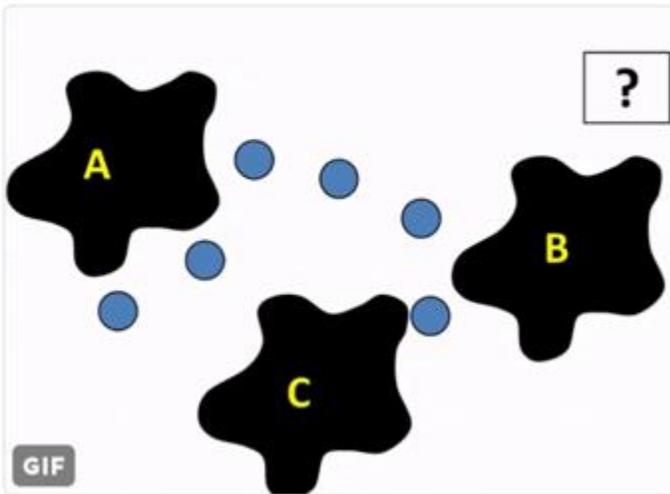
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# Motion Splats!

*\*Credit to Steve Wyborne and <http://www.stevewyborne.com/>*

## Problem:

They all begin with the same number...



Visit <http://www.stevewyborne.com/?p=1028> for Motion Splat activities.

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## Extension:

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## Notes:

[BACK](#)

## Target 6

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\*Credit to [http://mathsolutions.com/ms\\_classroom\\_lessons/the-game-of-target-300/](http://mathsolutions.com/ms_classroom_lessons/the-game-of-target-300/) and Math DDSB @Errs5

### Problem:

In this game, you will be multiplying by  $\frac{1}{2}$   $\frac{1}{3}$   $\frac{1}{4}$   $\frac{2}{3}$   $\frac{3}{4}$

Play this game with a partner.

You will each get 6 rolls of a die.

After each roll, you'll decide what number by which you will multiply.

The corresponding product will be your points for that roll.

Add your points from all 6 rolls.

The goal is to be the player closest to six at the end.

---

### Extension:

What if you use two dice?

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### Notes:

Visit: [http://mathsolutions.com/ms\\_classroom\\_lessons/the-game-of-target-300/](http://mathsolutions.com/ms_classroom_lessons/the-game-of-target-300/) to see the original game.

# Doubling the Dimensions

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*\*Credit to Alex Overwijk @AlexOverwijk*

## Problem:

What happens to the surface area and to the volume of a rectangular box when you double its dimensions?

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## Extension:

What if the box wasn't rectangular?

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## Notes:

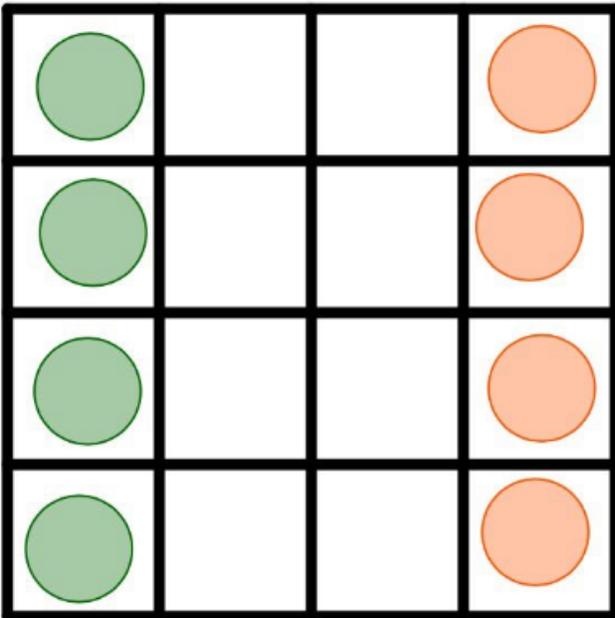
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## Bishops Switch Places

---

### Problem:

The green and orange counters can only move diagonally any number of spaces. There is no jumping or landing on top of each other. What is the fewest number of moves needed for the green and orange counters to switch places?



---

### Extension:

What if the grid was a different size?

---

**Notes:** Counters and blank grids would be helpful.

[BACK](#)

# A Knight's Tour

---

## Problem:

A knight's tour is a sequence of moves of a knight on a chessboard such that the knight visits every square only once. If the knight ends on a square that is one knight's move from the beginning square (so that it could tour the board again immediately, following the same path), the tour is *closed*, and otherwise it is *open*.

Can you find a closed knight's tour on an 8 by 8 chess board?

Can you find an open knight's tour on an 8 by 8 chess board?

---

## Extension:

What if the chessboard was a different size?

N by n?

---

## Notes:

Blank grids and counters.

Remember that sometimes it is easier to make a problem simpler. Start small.

Visit: [https://en.wikipedia.org/wiki/Knight%27s\\_tour](https://en.wikipedia.org/wiki/Knight%27s_tour)

# Tower Puzzles

---

*\*Credit to Real-Life Math Problem Solving by Mark Illingworth*

## Problem:

Katie built a tower with red blocks that was 2 blocks wide by 2 blocks long by 16 blocks tall. She claimed that he could take the tower apart and make a new one, using all the blocks, for which all three dimensions would be the same as each other. What are the dimensions of the new tower?

Katie made up a math puzzle for her 4 friends. She gave them each 64 blocks with which to build a rectangular tower that would appear square when viewed from above. How many different towers could they build? What tower could be built requiring the least amount of paint to cover its exposed sides?

---

## Extension:

Explore building towers with different dimensions.  
Make up your own tower puzzle.

---

## Notes:

Snapcubes.

## Fine Feathered Friend

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*\*Credit to Real-Life Math Problem Solving by Mark Illingworth*

### Problem:

Crows think they know everything, and they always have to be right. Baldwin, the crow that lives on our block, is the most arrogant of them all. Today I have to feed him half of my sandwich because I lost a bet with him. Baldwin asked me if I knew the expression “as the crow flies.” I told him I had heard it, but I didn’t know what it meant. He said that the shortest distance between two points is always as the crow flies – that is, a straight line. Our discussion ended in a bet. He said that he could give me a 3-block head start and still get to the sandwich shop before I could. Since we had raced before, I knew that I could ride my bike just as fast as Baldwin could fly. I took the bet. All our streets run north-south or east-west, and the shop is 6 blocks east and 8 blocks north. I lost both races. The first time, I rode directly east for 6 blocks and then directly north for 8 blocks. The next time I tried zig-zagging. Both times, he was laughing when I got there. Can you tell how many blocks Baldwin beat me by?

---

### Extension:

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### Notes:

Graph paper

[BACK](#)

# Big Nickel

\*Credit to @canada150math and [www.tapintoteenminds.com](http://www.tapintoteenminds.com)

**Problem:**



How many nickels?

---

**Extension:**

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**Notes:**

[BACK](#)

# Water Tank

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\*Credit to <http://threeacts.mrmeyer.com/watertank/>

## Problem:



Video here: <http://threeacts.mrmeyer.com/watertank/>

How long will it take the tank to fill up?  
Give an answer you know is too high.  
Give an answer you know is too low.  
Guess an answer as close as you can.

---

## Extension:

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## Notes:

Visit: [https://docs.google.com/spreadsheets/d/1jXSt\\_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0](https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0) for a bank of Dan Meyer's Three-Act Math Tasks.

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# Gold Coins

\*Credit to <https://brilliant.org/100day/day4/>

## Problem:

All of these coins have one gold side and one silver side. You are only allowed to flip over pairs of adjacent coins. For which of these arrangements of 9 coins is it possible to make the entire row gold?

Arrangement A



Arrangement B



## Extension:

Can you generalize your answer into a rule for when it's possible and when it's impossible to transform an arrangement to pure gold?

When it's possible, how can you do it in the fewest number of flips?

What if you could flip three adjacent coins instead of two?

## Notes:

Visit: <https://brilliant.org/explorations/joy-problem-solving/>

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## Doritos Roulette: Hot or Not?

---

*\*Credit to Kyle Pearce and tapintoteenminds.com*

### Problem:



How Many Chips are Hot?

<https://tapintoteenminds.com/3act-math/doritos-roulette-hot-or-not/#task>

Using the information on the bag, can you figure out around how many chips are in the bag?

If the ratio on the front of the bag is accurate, how many chips would you expect to be regular and how many would you expect to be hot? What percentage would be regular and what percentage would be hot?

---

### Extension:

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### Notes:

[BACK](#)

## Solve 1

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*\*Credit to H. Laurence Ridge, University of Toronto and Stephen I. Brown, State University of New York*

### Problem:

The sum of three consecutive integers is five more than the sum of the least and the greatest of the consecutive numbers. What are the numbers?

### Problem:

John is 19 years old and his sister Susan is only 1 year old. In how many years will John be:

- a) 7 times as old as Susan?
- b) 4 times as old as Susan?
- c) Only twice as old as Susan?
- d) The same age as Susan?

### Problem:

Brett's father is four times as old as Brett is now. In four years, Brett's age will be one-third of his father's age. How old are they now?

When will Brett be half his father's age?

---

### Extension:

Can you make your own Solve problems?

---

### Notes:

## Solve 2

---

*\*Credit to Stephen I. Brown, State University of New York*

### Problem:

If a perfect square is even, then the square root of that number is also even. Is this true? Can you prove it?

### Problem:

1x3

2x4

3x5

4x6

5x7

Explore! Look for connections. Look for patterns.

### Problem:

The ten's digit of a two digit number is one half the unit's digit. Four times the sum of the digits equals the number. Find the number.

---

### Extension:

Can you make your own Solve problems?

---

### Notes:

[BACK](#)

# Coins

---

*\*Credit to Stephen I. Brown, State University of New York*

## Problem:

Amy goes out to buy herself some candy. She has a bunch of change in her pocket. Reaching inside, she feels around and finds that:

- she has nickels, dimes and quarters
- there are 25 coins all together
- there are three more nickels than dimes
- the total amount of money is \$7.15

How many coins of each kind does she have?

## Problem:

John has some change in her purse. He has no dollar coins. He cannot make change for a nickel, a dime, a quarter, a half dollar, or a dollar. What is the greatest amount of money he can have?

## Problem:

Fran emptied his piggy bank and counted out exactly \$10 in nickels, dimes and quarters. There were twice as many dimes as nickels. There were twice as many quarters as dimes. How many nickels did she have?

---

## Extension:

Can you make your own coin problem?

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## Notes:

[BACK](#)

# Infinite Numbers

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## Problem:

There are infinite number between any two numbers. Explore and Prove!

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## Extension:

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## Notes:

# Build a Fence

---

*\*Credit to Al Anderson, Medicine Hat School System*

## Problem:

Can you build a fence so that my cows will have the most possible grass to eat?

---

## Extension:

What shapes did you try?

What if I had more fencing material?

---

## Notes:

Start with 16 base-10 rods or other manipulatives.

[BACK](#)

# Build Another Fence

---

*\*Credit to Al Anderson, Medicine Hat School System*

## Problem:



Mrs. Callahan is building a new dog pen for her dog, Cooper. She has 24 feet of fencing to build the dog pen. The pen needs to be a rectangle. What are the lengths and widths of the different sizes of pens that she can make with that amount of fencing? Which pen will give Cooper the largest area to run around in?

---

## Extension:

What shapes did you try?

What if I had more fencing material?

---

## Notes:

[BACK](#)

# Circles and More Circles

---

*\*Credit to Al Anderson, Medicine Hat School System*

## Problem:

What is the relationship of the distance around a circular object to the distance across it?

---

## Extension:

Graph the relationship.

Can you show the relationship using Cuisenaire rods?

---

## Notes:

Use 5-6 different size circular objects (cans, cups, chips, wheels).  
Scissors, string, ruler, paper or whiteboard.

Cuisenaire Rods

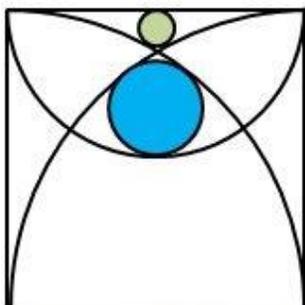
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# Circles Galore

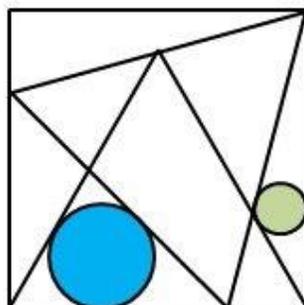
*\*Credit to San Gaku*

## Problem:

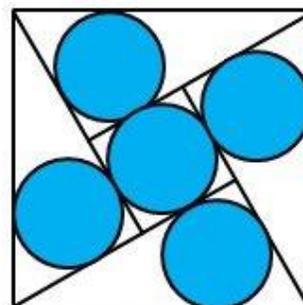
A selection of San Gaku problems



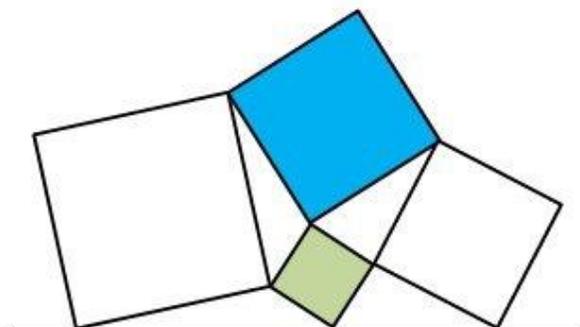
Find the relationship between the radii of the circles



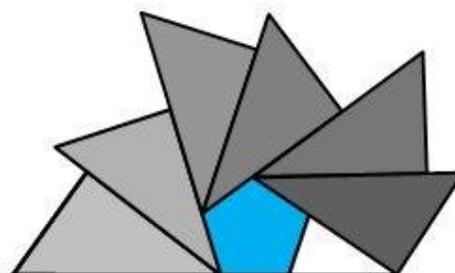
The triangles are equilateral. Find the relationship between the radii of the circles



Find the relationship between the side of the square and the radii of the congruent circles



Find the relationship between the sides of the coloured squares



Find the relationship between the hypotenuse of the congruent right-angled triangles and the side of the regular pentagon.

## Extension:

Can you make your own problem?

## Notes:

[BACK](#)

## Using Calculators 1

*\*Credit to Karen L. Jones, Charles E. Lamb, Frederick L. Silverman*

### Problem:

Consider two cars. One went 317.9km on 36.48 litres of gas; the other went 512.4km on 58.68 litres of gas. How do the cars compare in kilometres per litre?

### Problem:

Below is part of the record from a checking account. There is a \$1.90 discrepancy with the bank statement. Find any errors, and correct them. How did they occur? What should the balance be?

<u>Check #</u>	<u>Date</u>	<u>Amount</u>	<u>Previous Balance:</u>	<u>\$1245.18</u>
#1431	29 May	12.50	Bret's Hardward	1232.68
#1432	2 Jun	25.43	J.C. Peanuts, Inc.	1208.25
#1433	2 Jun	128.94	Post-Pine Furniture	1079.31
#1434	5 Jun	38.11	Mina Bird's Pets	1041.10
#1435	6 Jun	2.56	U.S.P.S.	1038.64
#1436	10 Jun	19.25	Henry's Hickory Hut	1019.39
#1437	11 Jun	29.87	Wonder Grocery	990.42

### Problem:

Use a calculator to find a pattern for the unit's digits in the sequence:

$$7^0, 7^1, 7^2, 7^3, 7^4 \dots$$

Try this activity with other base numbers.

### Extension:

Can you make your own Using Calculators problems?

### Notes:

[BACK](#)

## Using Calculators 2

---

*\*Credit to Karen L. Jones, Charles E. Lamb, Frederick L. Silverman*

### Problem:

How useful is the calculator in finding these products?

$$250 \times 10$$

$$267.5 \times 10$$

$$2750 \times 10$$

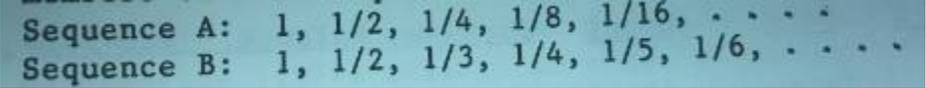
$$27.89 \times 10$$

Find a decimal representation for  $\frac{8}{15}$ . Is the calculator useful?

Find examples of calculations where the calculator is useful and examples of calculations where it is not useful.

### Problem:

Two sequences of numbers appear below. Investigate what happens when you add the same number of consecutive numbers of each sequence, starting at the beginning.



Sequence A: 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ , . . . .  
Sequence B: 1,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ , . . . .

---

### Extension:

Can you make your own Using Calculators problems?

---

### Notes:

[BACK](#)

## Using Calculators 3

---

*\*Credit to Karen L. Jones, Charles E. Lamb, Frederick L. Silverman*

### Problem:

A square has dimensions of 16 cm per side. If each side is halved, what effect is there on the area? Continue the process. What results emerge? Suppose you start with a 24cm square? Apply the same procedure. By what percentage does the area change?

### Problem:

Use the calculator to find the pattern for finding such products as those that follow:

$$15 \times 15$$

$$25 \times 25$$

$$35 \times 35$$

$$45 \times 45$$

$$55 \times 55$$

$$95 \times 95?$$
 Do you see an efficient way to find these products?

What about:  $15 \times 25$

$$25 \times 35$$

$$35 \times 45$$

$$45 \times 55$$

$$75 \times 85?$$
 Do you see an efficient way to find these products?

How does it compare to the way you found in the first part of this question?

---

### Extension:

Can you make your own Using Calculators problems?

---

### Notes:

[BACK](#)

# Pop Box Design

---

\*Credit to <http://mrpiccmath.weebly.com/blog/3-acts-pop-box-design>

## Problem:

Ever wonder why companies make the decisions that they do? My wife and I drink more pop than I am willing to admit, and one thing I noticed while at the store is that the twelve packs of Coke and Pepsi do not have the same design. Let's look at them.

Which one uses the least amount of cardboard?

Which company made the best choice for the environment?

Any other questions you want to investigate?

---

## Extension:

This is also a precursor to [this lesson](#).

---

## Notes:

Visit: <http://mrpiccmath.weebly.com/blog/3-acts-pop-box-design>

[BACK](#)

# Pop Can Factory

---

**Problem:**



This factory makes **1 450** cans of soda per minute.

What do you want to figure out?

---

**Extension:**

---

**Notes:**

[BACK](#)

# Visualizations

---

*\*Credit to Barbara Moses, Bowling Green State University*

## Problem:

A fireman stood on the middle step of a ladder, directing water into a burning building. As the smoke got less, he climbed up three steps and continued his work. The fire got worse so he had to go down five steps. Later, he climbed up the last six steps and was at the top of the ladder. How many steps were there? What if the ladder was longer? What if there was an even number of steps?

## Problem:

Sally had a new bike which she took to school every day. On some days she rides the bike to school and walks home. On the other days, she walks to school and rides the bike home. The round trip takes one hour. If she were to ride the bike both ways, it would only take  $\frac{1}{2}$  an hour. How long would it take if she walked both ways?

---

## Extension:

---

## Notes:

[BACK](#)

# Climbing

---

*\*Credit to Problem Solving in Mathematics, R.I.C. Publications*

## Problem:

Cathy Caterpillar wants to climb to the top of an 8-metre high tree. Each day she climbs forward 3 metres, but slips back 1 metre overnight. How long will it take her to reach the top of the tree?

## Problem:

Suzy Snail is climbing a steep 16-metre high rock wall. Each day she climbs forward 5 metres but slips back 2 metres overnight. How long will it take her to get to the top of the rock wall?

## Problem:

A well is 10 feet deep. Frank the Frog climbs up 5 feet during the day but falls back down 4 feet during the night. Assuming that the frog starts at the bottom of the well, on which day does he get to the top?

## Problem:

What if Inky Inchworm is climbing out of a glass that is 13 inches high and can climb  $1\frac{1}{2}$  inches in an hour but slides and falls back  $\frac{1}{4}$  inch during the hour it rests? How long will it take the snail to climb to the top of the jar?

---

## Extension:

Investigate different numbers, look for patterns, extend a pattern.

Can you make your own Climbing Problem?

---

## Notes:

[BACK](#)

## Who's Who?

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

### **Problem:**

Four married couples belong to a golf club. The wives' names are Kay, Sally, Joan, and Ann. Their husbands are Don, Bill, Gene, and Fred. Examine the following clues and decide who is married to whom:

- Bill is Joan's brother
- Joan and Fred were once a couple but didn't get married.
- Ann has two brothers, but her husband is an only child.
- Kay is married to Gene

### **Problem:**

Steve, Jim, and Calvin are married to Beth, Donna, and Jane, not necessarily in that order. Four of them are playing bridge. Steve's wife and Donna's husband are partners. Jane's husband and Beth are partners also. No married couples are partners. Jim does not play bridge. Who is married to whom?

### **Problem:**

Sue signed up for games at her school's fun night. Seven other people were assigned to her group, making up four pairs of partners. The other members of her group were Dave, Angie, Josh, Tanya, Joy, Stu, and Linus. When the games started, Dave and his partner were to the left of Stu. Across from Dave was Sue, who was to the right of Josh. Dave's brother's partner, Tanya, was across from Stu. Joy was not on Stu's right. Name the four pairs of partners.

---

### **Extension:**

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### **Notes:**

[BACK](#)

# Handshakes

---

**Problem:**

There are 12 people at a party. If everyone shakes hands with everyone else at the party, how many handshakes take place?

---

**Extension:**

What if there were more people? What if there were  $N$  people?

What if there were more handshakes?

---

**Notes:**

# The Honest Brothers

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

## Problem:

One of five brothers had broken a window. John said, "It was Henry or Thomas." Henry said, "Neither Earnest nor I did it." Thomas said, "You are both lying." David said, "No, one of them is speaking the truth, but not the other." Earnest said, "No, David, that is not true." Three of the brothers always tell the truth, but the other two cannot be relied on. Who broke the window?

---

## Extension:

---

## Notes:

Making a table is a useful strategy for this type of problem.

[BACK](#)

## Ordered Digits

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

### Problem:

0	1	2	3	4	5	6	7	8	9

In the ten cells above, inscribe a ten-digit number such that the digit in the first cell indicates the total number of zeros in the entire number; the digit in the cell marked 1 indicates the total numbers of ones in the number, and so on to the last cell, whose digit indicates the total number of nines in the number. Zero is a digit, of course. The answer is unique.

---

### Extension:

---

### Notes:

Making a table is a useful strategy for this type of problem.

[BACK](#)

## Circles of Students

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

### Problem:

Miss Young has her 18 students seated in a circle. They are evenly spaced and numbered in order. Which student is directly opposite: student number 1, student number 2, student number 18?

Mr. Evans seated his students in the same way as Miss Young's. Student number 5 is directly opposite number 26. How many students are in Mr. Evan's class?

Mrs. White teachers Phys. E. She had her students space themselves evenly around a circle and then count off. Student number 16 is directly opposite number 47. How many students are in Mrs. White's class?

---

### Extension:

A large number of people are standing in a circle and are evenly spaced. The 7<sup>th</sup> person is directly opposite the 791<sup>st</sup>. How many people are there altogether?

---

### Notes:

[BACK](#)

# Remainders and Divisors

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

## Problem:

Find the smallest number which, when divided by each of the integers 2, 3, 4, 5, 6, 7, 8, 9, and 10, will give, in each case, a remainder which is 1 less than the divisor.

---

## Extension:

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## Notes:

[BACK](#)

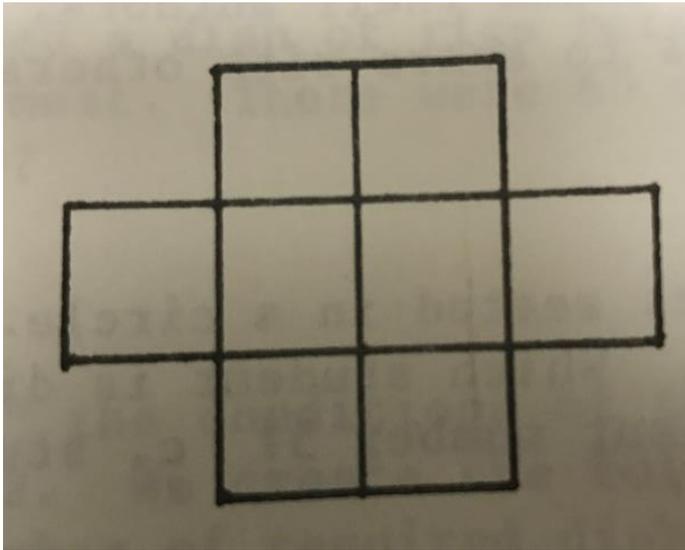
# One to Eight

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

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## Problem:

Fill in the following figure with the digits 1-8 in such a way that no two consecutive numbers are in boxes which touch at a point or side.



---

## Extension:

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## Notes:

[BACK](#)

# Exam

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

## Problem:

An exam has five true-false questions.

- a. There are more true than false answers.
- b. No three consecutive questions have the same answer.
- c. The students know the correct answer to problem number 2.
- d. Questions number 1 and number 5 have opposite answers. From the information above, the students were able to determine all the correct answers. What are they?

---

## Extension:

---

## Notes:

# Records

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

## Problem:

Jim has a collection of records. When he puts them in piles of two, he has one left over. He also has 1 left over when he puts them in piles of 3 or piles of 4. He has none left over when he puts them in piles of 7. What is the least number of records he may have?

---

## Extension:

What other numbers of records could he have?

---

## Notes:

# Christmas Gifts

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

## Problem:

It is traditional in many families at Christmas time for each family member to give a gift to each of the other members. How many gifts would be given if there were 10 family members? How about for your family which has \_\_\_\_ members?

---

## Extension:

---

## Notes:

## How Many? How Much?

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

### **Problem:**

If a clock strikes six times in five seconds, how many times will it strike in ten seconds?

### **Problem:**

How much will it cost to cut a log into eight equal segments, if cutting it into four equal segments costs 60 seconds?

### **Problem:**

The editor of your school yearbook knows that 2985 digits were used to print its page numbers. How many pages were in the yearbook?

---

### **Extension:**

---

### **Notes:**

## Trains and Highways

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

### **Problem:**

A certain highway was being repaired, so it was necessary for the traffic to use a detour. At a certain time, a car and truck met in this detour which was so narrow that neither the truck nor the car was able to pass. Now, the car had gone three times as far into the detour route as the truck had gone, but the truck would take three times as long to reach the point where the car was. If both the car and the truck can move backward at one third of their forward speed, which of these two vehicles should back up in order to permit both to travel through the detour in the minimum amount of time?

### **Problem:**

The shuttle service has a train going from Washington to New York City and from New York City to Washington every hour on the hour. The trip from one city to the other takes  $4\frac{1}{2}$  hours and all trains travel at the same speed. How many trains will pass you as you are going from Washington to New York City?

### **Problem:**

Herb was attempting to cross a railroad bridge. When he was  $\frac{3}{7}$  of the way across, he heard a train coming behind him. He ran to the far end and hopped off just as the train got to him. Later, he calculated that he could have run to the other end of the bridge and still have survived. If the train was going 35 kilometers per hour, how fast did Herb run?

---

### **Extension:**

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### **Notes:**

[BACK](#)

# Matchsticks

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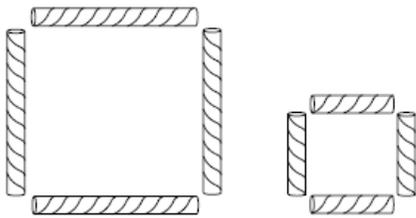
*\*Credit to Mathematical Challenges for Able Pupils*

## Problem:

### Square it up

You need six drinking straws each the same length.  
Cut two of them in half.  
You now have eight straws, four long and four short.

You can make 2 squares  
from the eight straws.



Arrange your eight straws to make 3 squares, all  
the same size.

---

## Extension:

How can 12 equal-length straws be arranged to make 6 regions of equal area?

---

## Notes:

[BACK](#)

# ABCDE

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*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

**Problem:**

$$\begin{array}{r} A B C D E \\ X \quad \quad 4 \\ \hline E D C B A \end{array}$$

Supply a digit for each letter so that the equation is correct. A given letter always represents the same digit.

---

**Extension:**

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**Notes:**

# Bank Robber

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*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

## Problem:

The security guard at a bank caught a bank robber. The robber, the teller, and a witness were arguing when the police arrived. This was what the police learned in the confusion:

- a. The names of the 3 men were Brown, Jones, and Smith.
- b. Brown was the oldest of the three.
- c. The teller and Jones had been friends for many years.
- d. Brown was the brother-in-law of the witness.
- e. Smith graduated from high school 5 years earlier than the robber.

Who was the robber? Who was the teller? Who was the witness?

---

## Extension:

---

## Notes:

# Survey

---

*\*Credit to Cheryl Kantecki and Lee E. Yunker, West Chicago Community High School*

## Problem:

In a survey of 25 college students at the University of Calgary, it was found that of the 3 newspapers, Calgary Herald, Calgary Sun, and the Globe and Mail, 12 read the Herald, 11 read the Sun, 10 read the Globe and Mail, 4 read the Herald and the Sun, 3 read the Herald and the Globe and Mail, 3 read the Sun and the Globe and Mail, and 1 person read all 3.

What do you want to know? What do you want to find out?

---

## Extension:

How many read none of the newspapers?  
How many only read the Globe and Mail?

What if the ratios stayed the same but there were more students? How many newspapers would be read?

---

## Notes:

[BACK](#)

## Ages of Three Children

---

*\*A Classic! \*Credit to Dr. Mike Shaughnessy's class Visual Math for Middle School Teachers, 1995*

### Problem:

The host at a party turned to a guest and said, "I have three daughters and I will tell you how old they are. The product of their ages is 36. The sum of their ages is my house number. How old is each?" The guest rushed to the door, looked at the house number, and informed the host that he needed more information. The host then added, "The oldest likes strawberry pudding." The guest then announced the ages of the girls. What are the ages of the three daughters?

### Problem:

A census taker approaches a woman leaning on her gate and asks about her children. She says, "I have three children and the product of their ages is seventy-two. The sum of their ages is the number on this gate." The census taker does some calculation and claims not to have enough information. The woman enters her house, but before shutting the door tells the census taker, "I have to see to my eldest child who is in bed with measles." The census taker departs, satisfied.

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### Extension:

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### Notes:

All ages are whole numbers. It is possible that there may be twins or triplets.

There are variations of this problem.

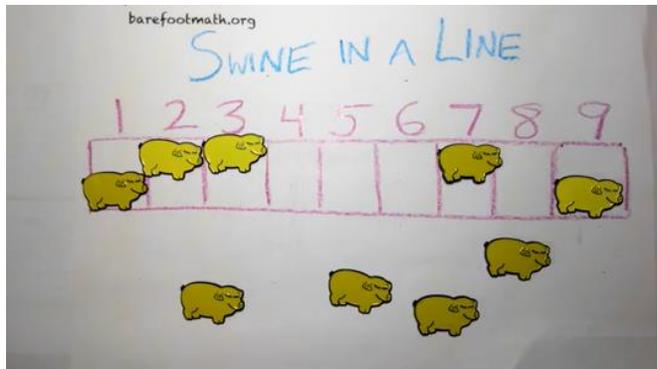
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## Swine in a Line

\*Credit to James Propp @jimpropp and barefootmath.com

### Problem:

Watch this video and figure out a winning strategy: <https://youtu.be/epbmEXr1o6s>



There are 9 pigs and a row of 9 pig pens that we want to put the pigs into. Two players take turns putting pigs into pens until every pen has exactly one pig in it. The player that puts that last pig into a pen wins. You can put a pig into an empty pen or you can put a pig in a pen that already has one pig in it. If you add a pig to a pen that already has a pig in it, one of the pigs jumps to the left, the other jumps to the right. This continues until every pen has at most one pig in it. When a pig jumps to the left from the far left or to the right from the far right, the pig just goes back to wandering in the field.

If there are pigs in pens 2, 7, and 9, what's the winning move? There is only one!

### Extension:

What if the pigs started in different pens?

### Notes:

Whiteboards, markers, and manipulatives!

[BACK](#)

# Tree Farm

---

**Problem:**

Each orange tree grown in California produces 720 oranges per year if not more than 20 trees are planted per acre. For each additional tree planted per acre, the yield per tree decreases by 15 oranges due to overcrowding.

---

**Extension:**

How many trees per acre should be planted to obtain the greatest number of oranges?

---

**Notes:**

Graph it!

Visit [here](#) for more.

# Sprinkler

---

*\*Credit to Fawn Nguyen*

## Problem:

A rectangular lawn has an area of 667 square meters. Surrounding the lawn is a flower border 4 meters wide. The border alone has an area of 548 square meters. A circular sprinkler is installed in the middle of the lawn.

What is the spraying radius of the sprinkler if it covers the entire yard, including the flower border?

What percentage of water is wasted?



---

## Extension:

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## Notes:

[BACK](#)

# Ration Ratios

---

*\*Credit to Fawn Nguyen*

## Problem:

At a recent math conference, lunch was provided for the participants. To be sure that there was enough food for everyone, the kitchen staff made more lunches than there were people attending. In fact, the ratio of prepared lunches to people was 7:5.

Because they anticipated a large number of vegetarians at the conference, the staff made 2 vegetarian lunches for every 3 non-vegetarian lunches.

It turned out that the ratio of non-vegetarians to vegetarians at the conference was 3:4.

What was the ratio of vegetarian lunches to vegetarians?

---

## Extension:

---

## Notes:

[BACK](#)

# Dollars Grow

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

Your aunt will give you \$1000.00 if you invest it for 10 years in an account that pays 20% interest compounded annually. That is, at the end of each year, your interest will be added to your account and invested at 20%. How will your money grow?

---

## Extension:

What if it were different amounts of money and interest? What if it wasn't compounded annually?

---

## Notes:

[BACK](#)

# Cutting the Cake

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

What is the maximum number of pieces you can cut a cake into by making four straight cuts with a knife? Each cut must pass through the top and bottom of the cake.

---

## Extension:

What is the maximum number of pieces into which a cake can be cut with five straight cuts?

---

## Notes:

Make the problem simpler to start.

[BACK](#)

# Square Share

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

Lines can divide squares into smaller squares. How many lines would I need to divide a square into 100 smaller squares? Into 400 smaller squares?

---

## Extension:

How many lines would I need to divide a square into  $n$  smaller squares?

---

## Notes:

Make the problem simpler to start. Table is helpful. Look for a pattern.

[BACK](#)

# Triangle's Turn

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

Three lines can divide an equilateral triangle into four smaller equilateral triangles. How? How many lines divide an equilateral triangle into nine smaller triangles? Into 25 smaller triangles?

---

## Extension:

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## Notes:

[BACK](#)

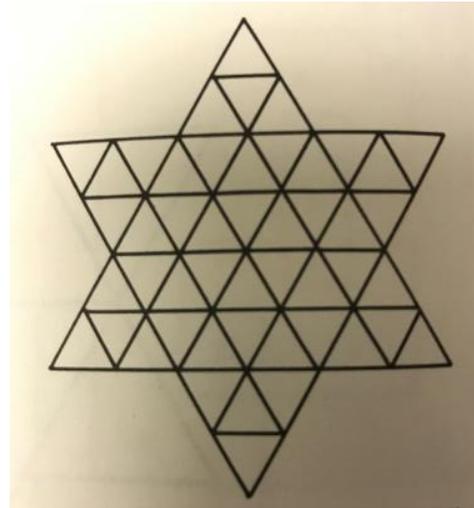
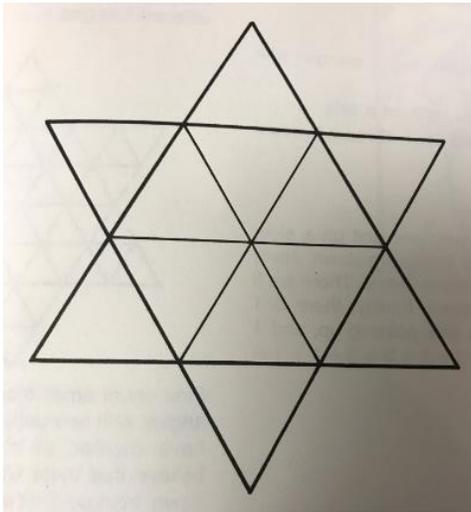
# Star Stitch

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

How many triangles are in each of these stars?



How can you be sure?

---

## Extension:

What if it was a different size star? Can you find a pattern?

---

## Notes:

[BACK](#)

# Polygon Parade

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

Show how four identical isosceles right triangles can be placed together to form:

- A rectangle that isn't square
- Two different parallelograms that aren't rectangles
- A square
- A triangle
- A trapezoid

---

## Extension:

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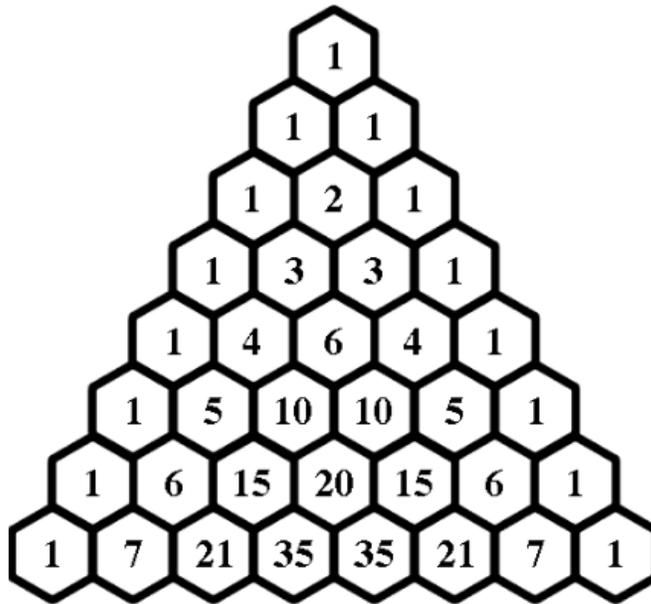
## Notes:

## Pascal's Triangle

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### Problem:

The triangle shown below is called Pascal's Triangle. What numbers are in the next rows of this triangle? What patterns can you find?



---

### Extension:

---

### Notes:

[BACK](#)

# Bounce

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

My rubber ball bounces exactly half the height from which it is dropped. I drop the ball from the top of a building that is 64 meters tall. How high will the ball bounce on its eighth bounce?

---

## Extension:

What if it kept on bouncing?

---

## Notes:

# Goldbach's Conjecture

---

*\*Unsolved math problem*

## Problem:

Think of any number. Now find the prime numbers that add up to that number. Does it work for every number?

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics.

---

## Extension:

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## Notes:

[https://en.wikipedia.org/wiki/Goldbach%27s\\_conjecture](https://en.wikipedia.org/wiki/Goldbach%27s_conjecture)

[BACK](#)

## Mrs. Times

---

*\*Credit to Rina Zazkis*

### Problem:

Mrs. Times went grocery shopping. She bought 6 bottles of coke for \$1.75 each and 5 kg of apples for \$2.40 per kg. She also bought some lemons for 60 cents each. The cashier counted the lemons and said your total is \$27.50 please. "There must be a mistake," Mrs. Times immediately said. How did she know?

---

### Extension:

---

### Notes:

[BACK](#)

# Wardrobe

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

My favourite clothes include 4 T-shirts, three pairs of jeans, and two pairs of sandals. How many days in a row could I wear a different outfit using my favourite clothes?

---

## Extension:

What if I had more favourite clothes?

---

## Notes:

[BACK](#)

## Red and White Cube

---

*\*Credit to Dale Seymour and Favorite Problems*

### **Problem:**

A 3 by 3 by 3 cube can be built with red cubes and white cubes so that no white faces touch and no red faces touch. How would you do this? How many small red faces and how many small white faces will be visible on the large cube?

---

### **Extension:**

Will edges of the same colour touch?

What if it was a bigger cube?

---

### **Notes:**

# One-Two-Three

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

How many different two-digit numbers and how many different three-digit numbers can be written using the digits 1, 2, and 3? Each digit can only be used once in a number.

---

## Extension:

What if each digit could be used more than once in a number?

What if you were also allowed to use the digit 4?

What if it was a four-digit number?

---

## Notes:

[BACK](#)

# Fool's Gold

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

A jeweler has four small bars that are supposed to be gold. He knows that one is counterfeit. The counterfeit bar has a slightly different weight than a real gold bar. Using only a balance scale, how can the jeweler find the counterfeit bar?

---

## Extension:

What if there was one counterfeit bar in 64 bars?

---

## Notes:

[BACK](#)

# Four Weights

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

Using a balance scale, you must be able to balance every whole kilogram amount from 1 kg through 15 kg. You may choose four standard weights to use, each a different number of kilograms. Which weights should you choose?

---

## Extension:

Suppose you wanted to balance every whole kilogram amount from 1kg through 31kg. Which five weights would you choose?

---

## Notes:

[BACK](#)

## Fast Draw

---

*\*Credit to Dale Seymour and Favorite Problems*

### Problem:

Form exactly two squares by drawing five lines. By drawing six lines. By drawing seven lines.

---

### Extension:

Can you draw two equilateral triangles using four lines?

Can you draw two equilateral triangles using five lines?

---

### Notes:

[BACK](#)

# Fundamental Theorem

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

The Fundamental Theorem of Arithmetic states that every natural number can be expressed as the product of prime numbers in only one way.

Do you think this is true?

Can you prove it?

---

## Extension:

---

## Notes:

# Dice Thrice

---

*\*Credit to Dale Seymour and Favorite Problems*

## Problem:

Rolling three regular dice, what is the smallest sum that could be rolled? What is the largest sum? How many different ways could a sum of ten be rolled?

---

## Extension:

---

## Notes:

Use three different coloured dice.

[BACK](#)

## Black and White Tiles

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### Problem:

A square floor is tiled with black and white square tiles. The tiles on the two diagonals are black. The rest of the tiles are white. If there are 101 black tiles in the floor, what is the total number of tiles in the floor?

---

### Extension:

What if there were a different number of tiles?

---

### Notes:

[BACK](#)

# Splitting Sixty Four

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

## Problem:

Sixty-four can be split into four parts so that when 3 is added to the first part, 3 is subtracted from the second part, the third part is multiplied by 3, and the fourth part is divided by 3, all four parts will be equal. What are the four parts?

---

## Extension:

---

## Notes:

Manipulatives, make a table.

[BACK](#)

## Waiting in Line

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### **Problem:**

The week before a big concert, five friends, Alvin, Bert, Chip, Dennis, and Ethan stood in line to buy tickets. There were other people between them, but the five friends were in the line in the order listed above. The line was really long, and they were pretty bored. Chip noticed that if he counted the number of strangers between each pair of friends, he got the numbers 7, 12, 19, 21, 36, 43, 55, 57, 64, and 76. He sent this information to Ethan in a paper airplane, and Ethan passed the time in line using Chip's information to determine how many people were between Alvin and Bert, between Bert and Chip, between Chip and Dennis, and between Dennis and Ethan. What did he find?

---

### **Extension:**

Is there more than one solution?

---

### **Notes:**

[BACK](#)

# Bean Bags

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

## Problem:

Lima beans come in 3 pound and 5 pound bags which cost \$1.15 and \$1.63 respectively. How many of each should you buy to have at least 17 pounds of lima beans at the lowest cost?

---

## Extension:

What if you wanted less lima beans? What if you wanted more lima beans?

---

## Notes:

[BACK](#)

# Wrong Number

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

## Problem:

Debbie forgot the last two digits of Linda's phone number. Debbie is determined to get in touch with Linda. "I'll just dial phone numbers until I reach her. There can't be that many, can there?" How many wrong numbers can she dial?

---

## Extension:

What if she forgot the last 3 digits? What about the last 4 digits?

How many 7-digit phone numbers could she dial if she didn't know any of the digits?

---

## Notes:

[BACK](#)

# Souvenirs

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

## Problem:

Tracy had saved her babysitting money for a month so that she could buy some souvenirs while on vacation in Mexico. She promised her best friends that she would bring each of them a gift from her trip. As soon as she arrived, she spent one fifth of her money to buy a T-shirt for Greta. The next morning she bought a hat for Chris that cost one fourth of the money she had left. The next day, she saw a cute bracelet for Tess, so she spent one third of her remaining funds. On the last day of her trip, she wanted to get something for herself. She spent half of what she had left on a sweatshirt she really liked. She went home with \$12 in her wallet. How much money did she start with? Which souvenir was the most expensive?

---

## Extension:

---

## Notes:

## Fair Shares

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### Problem:

Tom, Ted, Tony, and Terry worked together on a job. The job paid \$500. Each boy was paid according to the amount of work he contributed to the job. Ted and Tony earned \$280 working together. Ted and Tom have \$260, and Ted and Terry got \$220. What percent of the job did each boy do?

---

### Extension:

---

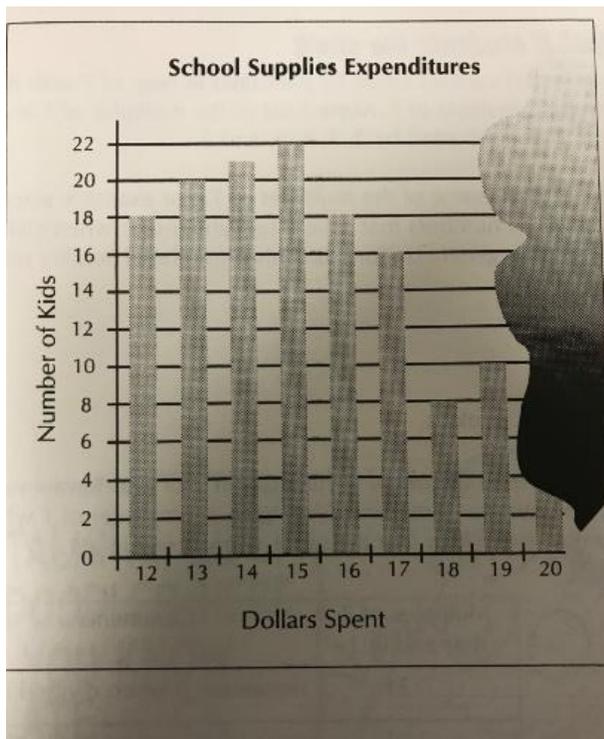
### Notes:

## Graph Goof

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### Problem:

Tino conducted a survey of all the eighth graders in his school for his math project. He asked each student how much he or she spent on school supplies at the beginning of the year (rounded to the nearest dollar). He calculated the average amount spent and found it to be \$15.50. Unfortunately, it rained on the day he brought the graph to school, and the last bar got smudged... Can you fix it?



### Extension:

<http://graphingstories.com/> for more activities on analyzing graphs.

### Notes:

[BACK](#)

## 3-D Cubes

---

*\*Credit to P. Harrison and [www.bovinemath.com](http://www.bovinemath.com)*

### **Problem:**

<http://www.bovinemath.com/cubes/documents/GETTING%20STARTED.pdf>

---

### **Extension:**

---

### **Notes:**

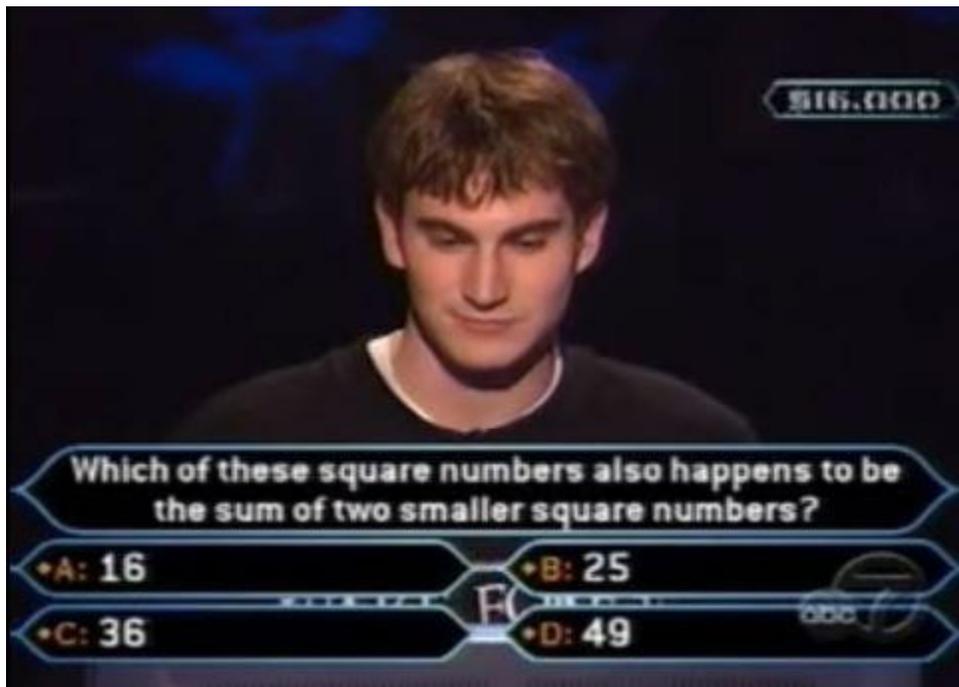
[BACK](#)

## Will it Cost You?

---

Problem:

<https://m.youtube.com/watch?v=BbX44YSsQ2I>



---

Extension:

---

Notes:

[BACK](#)

## A Ridiculous Ruler

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### Problem:

Mr. Oakley, my woodshop teacher, is kind of strange. He has a 12-inch strip of wood with only four marks on it that he uses as a ruler. Yesterday I asked him about it. "Mr. Oakley," I said, "What good is that ruler you use if it only has four marks on it? There are some lengths you can't measure!"

"Nonsense!" he replied, "I have placed the marks in such a way that I can measure any whole number length from 1 to 12 inches between the marks. I think you should make one for yourself!"

Can you make one for yourself?

---

### Extension:

What if you needed a ruler that was longer than 12 inches?

---

### Notes:

[BACK](#)

## Split the Cost

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### Problem:

The cost of chartering a bus for the ski club's annual winter trip was split evenly by the 20 students planning to go. The day before the trip, 10 more students decided to go. Dividing the cost evenly among all 30 students brought the average cost down by \$9.00 per person. What was the total cost of chartering the bus?

---

### Extension:

---

### Notes:

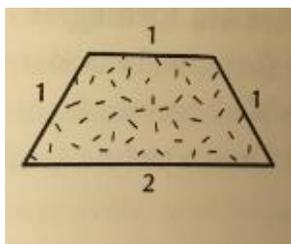
[BACK](#)

## Trapezoid Challenge

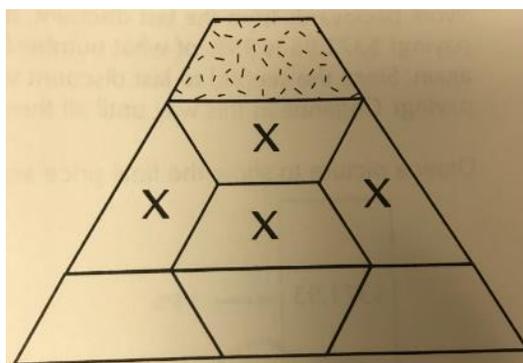
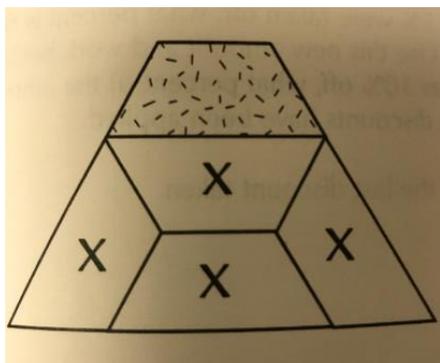
*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### Problem:

This trapezoid has an area of one and a perimeter of five.



Using more trapezoids just like this one, it is possible to build larger and larger trapezoids by alternating, adding a “thick” layer, followed by a “thin” layer at the base of the trapezoid.



Find the perimeter of a large trapezoid with an area of 96.

### Extension:

What other patterns can you find or investigate?

### Notes:

Trapezoid fraction blocks.

[BACK](#)

## What a Deal

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### Problem:

Sam went to TV City to buy a new TV. When he got there, he found that the manager was offering a 25% discount on every TV in the store. The one Sam wanted was last year's model so he got an additional 10% discount on his purchase. When he got to the cash register, he found out he would save even more because the store offered a 5% discount for paying cash. These discounts were applied successively, and he paid \$371.93 for the TV. What was the original price of the TV? What was the total percent discount he got?

---

### Extension:

---

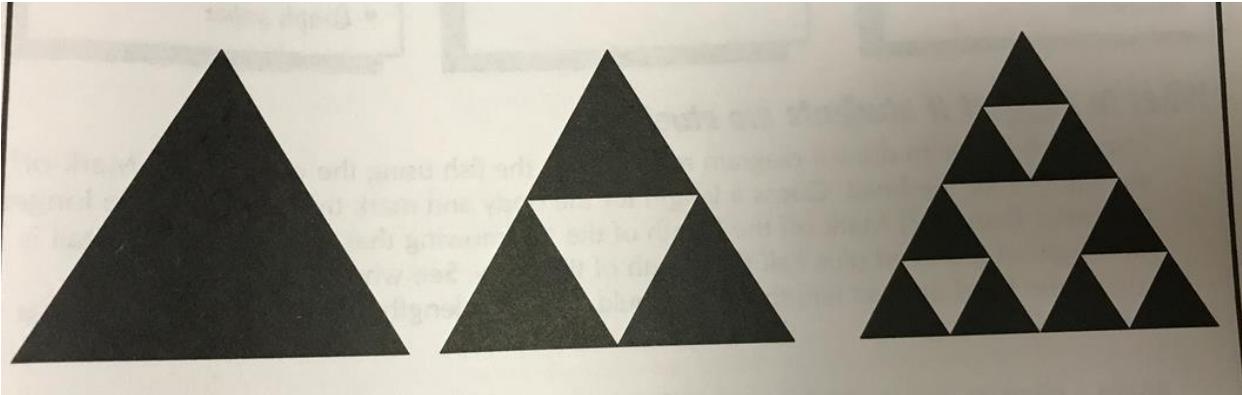
### Notes:

[BACK](#)

# Tricky Triangles

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

## Problem:



A large triangle changes each day as shown. Each day a white triangle appears in the center of each shaded one. If this pattern continues, how many white triangles will there be on the sixth day? On the sixth day, what fraction of the outer triangle will be white?

---

## Extension:

How far can you extend this pattern by drawing it or by building it?

Can you build a 3-D version?

---

## Notes:

Explore Sierpinski's triangle.

Triangular grid paper.

[BACK](#)

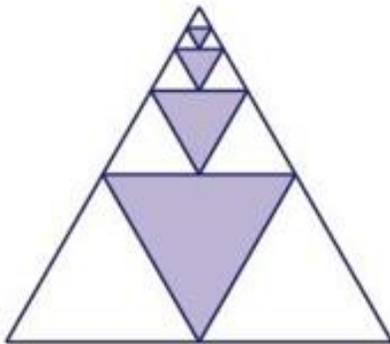
# Trickier Triangles

---

\*Credit to Maths Medecine [www.mathsmed.co.uk](http://www.mathsmed.co.uk)

## Problem:

What fraction of this triangle is shaded if the pattern goes on forever? The biggest shaded triangle is  $\frac{1}{4}$  of the white triangle.



---

## Extension:

How far can you extend this pattern with fractions? What sum are you approaching?

---

## Notes:

Explore Sierpinski's triangle.

Triangular grid paper.

[BACK](#)

# Science Squares

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

## Problem:

My science class is pretty small. There are just 18 students in the class. My teacher, Ms. Microbe, has an unusual system for pairing us up for labs. She has given each of us a number from 1 through 18, and on lab days, she pulls our numbers out of a bag to determine who is paired with whom. When we did our last lab, I happened to notice that the sum of each person's number with his/her lab partner's number was a perfect square!

How were the partners assigned?

---

## Extension:

What is the probability of this happening for one pair, two pair, more? All of them?

---

## Notes:

[BACK](#)

# To Swim or Not to Swim

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

## Problem:

Ms. Poole, my P.E. teacher, asked the 100 seventh graders in my school if they could swim. Half the boys and two thirds of the girls said yes. Seventeen boys said no. How many seventh graders know how to swim?

---

## Extension:

What if there were more students?

---

## Notes:

[BACK](#)

# Tennis Anyone?

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

## Problem:

Seventy-eight players entered a single elimination tennis tournament. How many matches were played to determine the overall champion?

---

## Extension:

What if there was a different number of players?

What if it took two losses to be eliminated?

Investigate different tournament set-ups in sports.

What schedule could an organizer write for a six-team volleyball tournament so that each team will play every other team once and no team will ever be idle?

---

## Notes:

# Cycling Shop

---

*\*Credit to Teaching Children Mathematics, August 2016*

## Problem:



Imagine you work at a cycling shop building unicycles, bicycles, and tricycles for customers. One day, you receive a shipment of 8 wheels. Presuming that each cycle uses the same type and size of wheel, what are all the combinations of cycles you can make using all 8 wheels?

---

## Extension:

What if you had more wheels?

---

## Notes:

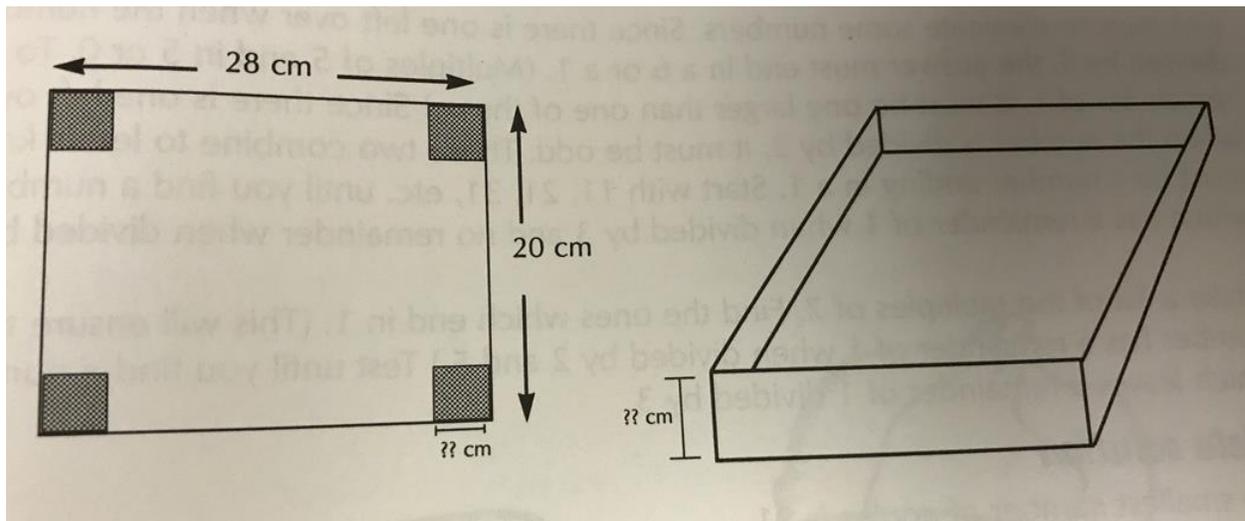
[BACK](#)

## Boxed In

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

### Problem:

Amy's project in metal shop class is to make a box with no lid. Her shop teacher gave her a piece of metal that is 28cm long and 20cm wide. The assignment is to form the box by cutting squares out of each corner and bending the sides up as shown below. The shop teacher, Mr. Rust, will give extra credit to any student who forms the box that holds the most. How large a square should Amy cut from each corner to form the box with the greatest volume?



### Extension:

### Notes:

[BACK](#)

# Chucky Cheese

\*Credit to John Orr @MrOrr\_geek

## Problem:



What is the value of a game token?

What's the price of a large pizza with 4 drinks?

How much do the cheesy breadsticks cost?

## Extension:

## Notes:

[BACK](#)

## Connecting Dots on a Circle

---

### Problem:

How many lines do we need to connect 2 dots on a circle? 3 dots on a circle? 4 dots on a circle? More dots on a circle?

---

### Extension:

$N$  dots on a circle?

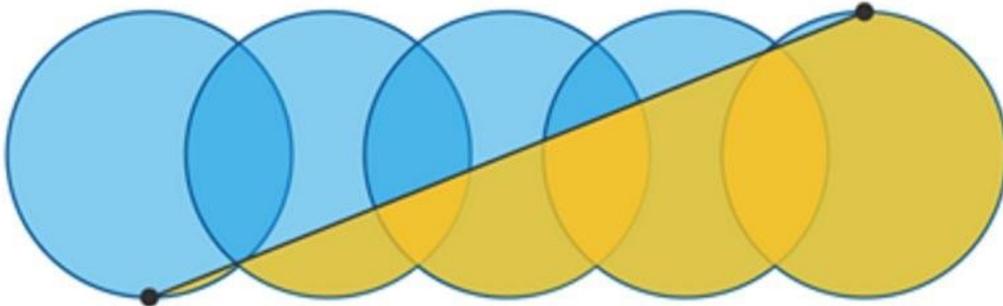
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### Notes:

## Overlapping Circles

---

**Problem:**



Each of the four overlapping regions of two circles has an area of 5, and the gold an area of 35.

**What is the area of one of the circles?**

---

**Extension:**

---

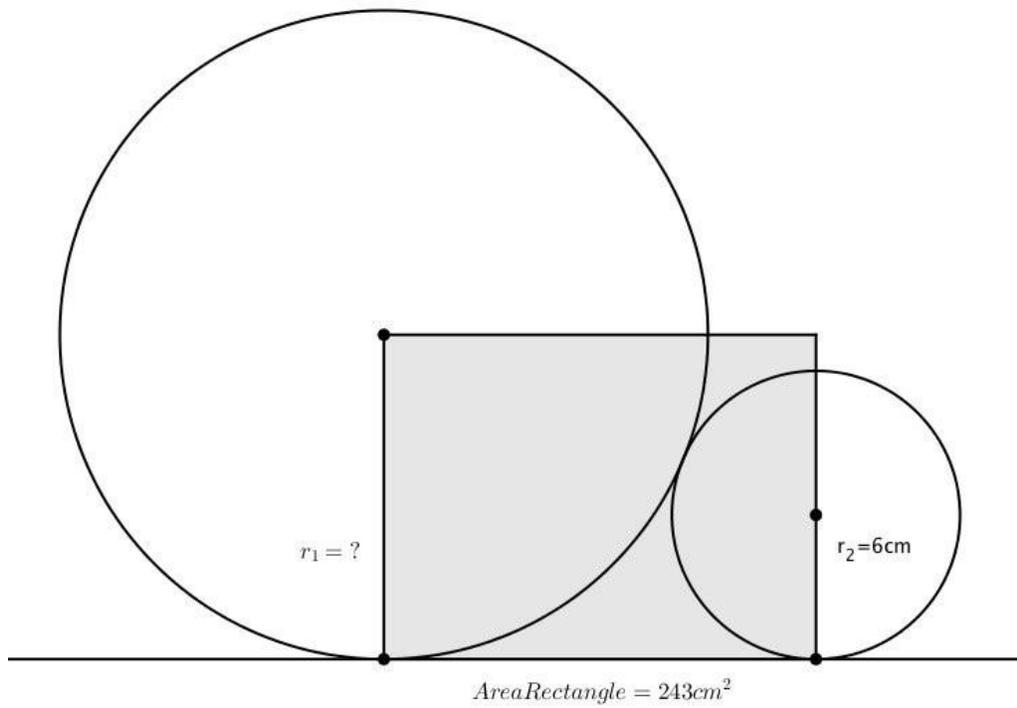
**Notes:**

[BACK](#)

# Adjacent Circles

---

**Problem:**



---

**Extension:**

---

**Notes:**

[BACK](#)

# Toast

---

*\*Credit to Thinking Mathematically by John Mason*

## Problem:

Three slices of bread are to be toasted under a grill. The grill can hold two slices at once but only one side is toasted at a time. It takes 30 seconds to toast one side of a piece of bread, 5 seconds to put a piece in or take a piece out, and 3 seconds to turn a piece over. What is the shortest amount of time in which the three slices can be toasted?

The answer is not 151 seconds!

---

## Extension:

What about 4 slices? 5 slices? More slices?

---

## Notes:

[BACK](#)

## Fraction Tricks

---

### Problem:

$$\frac{3}{5} + \frac{1}{5}$$

Add the numerators and keep the denominators.

$$2\frac{3}{5} = \frac{x}{5}$$

Multiply and add.

$$\frac{3}{5} \times \frac{1}{5}$$

Multiply straight across.

$$\frac{3}{5} = \frac{6}{10}$$

Multiply the top and bottom by the same number.

$$\frac{3}{5} \div \frac{1}{5}$$

Flip and multiply.

$$\frac{3}{5} = \frac{x}{5}$$

Cross multiply.

$3 \times 10$   
Move the decimal to the right.

$\frac{3}{10}$   
Move the decimal to the left.

$\frac{x}{5} = 2$   
Multiply both sides by the same number.

Pick a trick. Explain why it works. Does it always work?

Do you like it? Does it help you understand?

---

### Extension:

Can you find another trick to analyze?

---

### Notes:

[BACK](#)

## Cross Multiply

---

**Problem:**

$$\frac{x}{4} = \frac{6}{5}$$

$$5x = 24$$

$$x = \frac{24}{5}$$

Cross Multiplication (or cross multiply and divide).

Why do we use it? When do we use it?

How does it work? Why does it work?

Does it help you understand?

---

**Extension:**

Can you find another trick to analyze?

---

**Notes:**

## 8 Point Square

---

*\*Credit to Corinne Angier @atmcorinne*

### **Problem:**

Take an 8 point square (with points on all 3 corners and at the midpoint of each side length). Draw two lines so that you don't get any halves, quarters, or eighths.

---

### **Extension:**

---

### **Notes:**

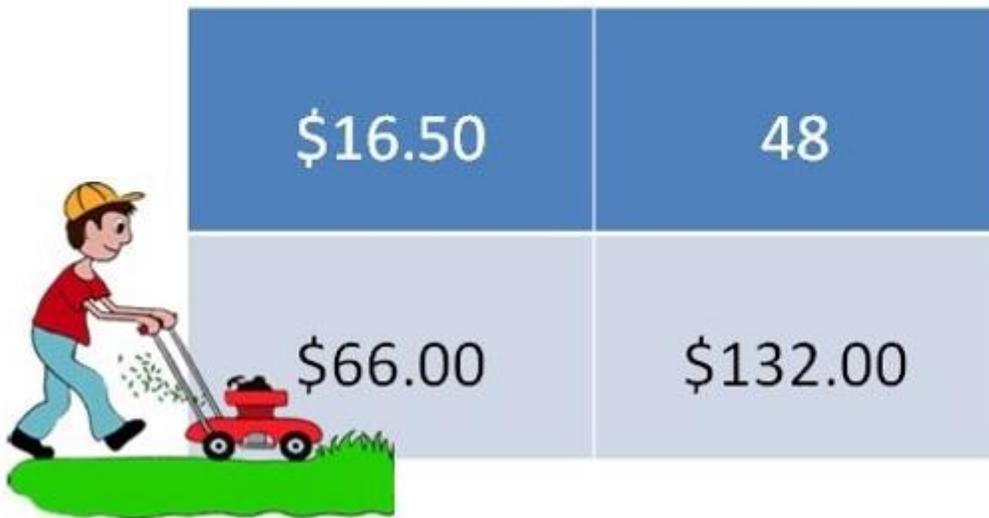
[BACK](#)

## What is the Question?

---

### Problem:

Joe earned \$792 for 12 days of work. Each day, he worked for 4 hours. What is the question if the answer is...



---

### Extension:

Can you invent your own "What is the Question" problem?

---

### Notes:

[BACK](#)

# Puck Play

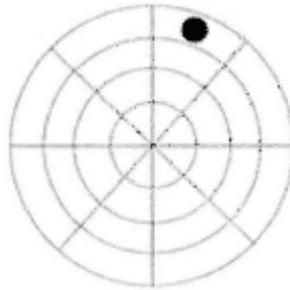
---

*\*Credit to Richard Hoshino*

## Problem:

Ferin and Ian play a game by alternately moving a hockey puck on a board with  $n$  concentric circles divided into  $r$  regions. For example, in the diagram below, we have  $n=4$  and  $r=8$ . The game starts with the puck already on the board, as shown. A player may move either clockwise one position or one position towards the centre, but cannot move to a position that has been previously occupied. The last person who is able to move wins the game.

Ferin moves first. Can Ferrin win?



---

## Extension:

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## Notes:

[BACK](#)

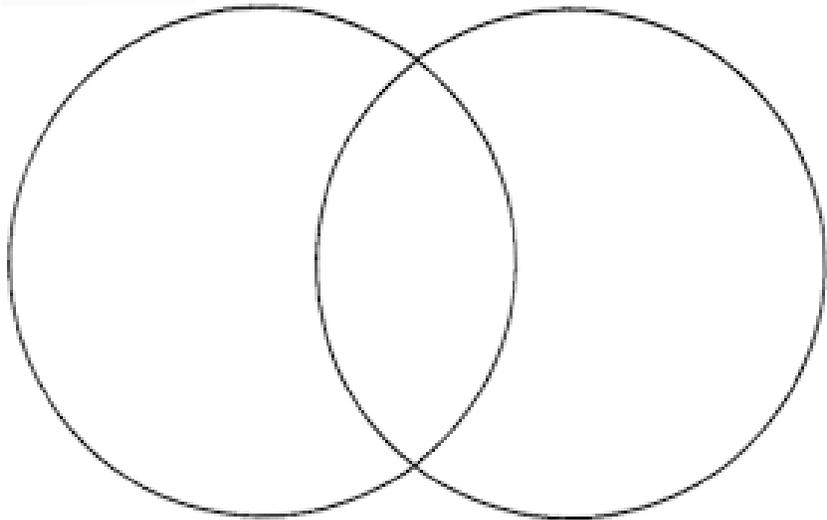
# Intersecting Sets

---

*\*Credit to Peter Liljedahl*

## Problem:

Place 17 objects into the two circles below so that each circle has the same number of objects. How many ways can you do it?



What if you want the ratio of objects in one circle to the other circle to be 4:3 or 12:9?

---

## Extension:

---

## Notes:

[BACK](#)

## 4 numbers

---

### Problem:

Pick 4 numbers such that there is one in each row and each column. Add them together. What do you notice? What do you wonder? Why?

1	2	3	4
5	6	7	8
9	10	11	12

---

### Extension:

---

### Notes:

[BACK](#)

# Eleven

---

*\*Credit to Problem of the Week, Jane Fesler*

## Problem:

Jonathan hated to empty the trash. He whined and complained and made up lots of excuses, but his mom still made him empty it because it was his job. Finally, one day his mom got tired of listening to him and made him a deal. She said, "You can be excused from emptying the trash one time for each solution you can find to this puzzle."

I am a three-digit number  
I am made up of three different numerals.  
I am divisible by eleven.

If you were Jonathan, how many times would you be excused from emptying the trash?

---

## Extension:

What if I was a four-digit number?

---

## Notes:

[BACK](#)

# Six Team Tourney

---

*\*Credit to Problem of the Week, Jane Fesler*

## Problem:

What schedule could an organizer write for a six-team baseball tournament so that each team will play every other team once and no team will ever be idle?

---

## Extension:

How many games would have to be scheduled if a new team enters the tournament?

Is there a way to tell how many games will need to be scheduled for a tournament, no matter how many teams want to play?

---

## Notes:

[BACK](#)

# Art Class

---

*\*Credit to Problem of the Week, Jane Fesler*

## Problem:

Alex is taking an art class where he is learning to paint pictures. The pictures get a little more complicated as the class progressed. Alex paints a picture. Then one minute later he paints another picture. Two minutes later he paints another picture. Four minutes later he paints another picture. Eight minutes later, he paints another picture, and so on. Alex's art teacher was really impressed with Alex's work and told him that he could have a special show to display his work as soon as he completed 25 pictures. How long will it take Alex to paint enough paintings so he can have his special show?

---

## Extension:

What if he had to finish a different number of paintings?

How many paintings could he finish in an hour?

---

## Notes:

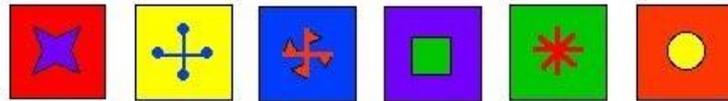
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## Faces of a Cube

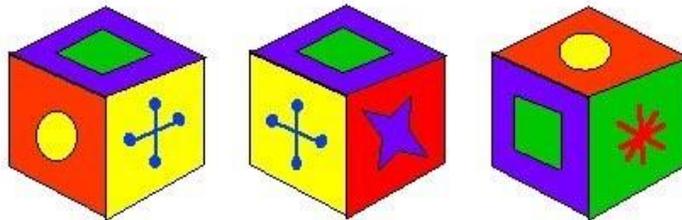
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### Problem:

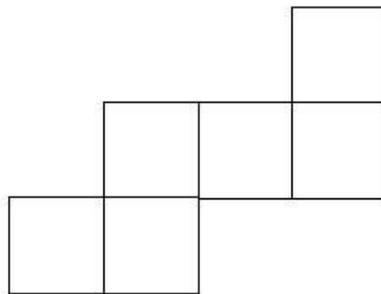
Here are the six faces of a cube - in no particular order:



Here are three views of the cube:



Can you deduce where the faces are in relation to each other and record them on the net of this cube?




---

### Extension:

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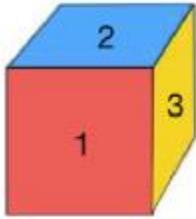
### Notes:

## More Faces of a Cube

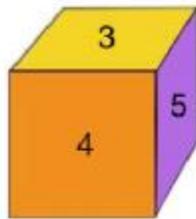
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*\*Credit to Brilliant.org*

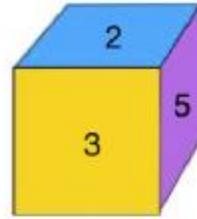
### Problem:



Perspective 1



Perspective 2



Perspective 3

In the above image, a cube is painted so that each side is a different number. If each of the colors 1, 2, 3, 4, 5, and 6 are to be used, then what side is opposite the side colored 2?

---

### Extension:

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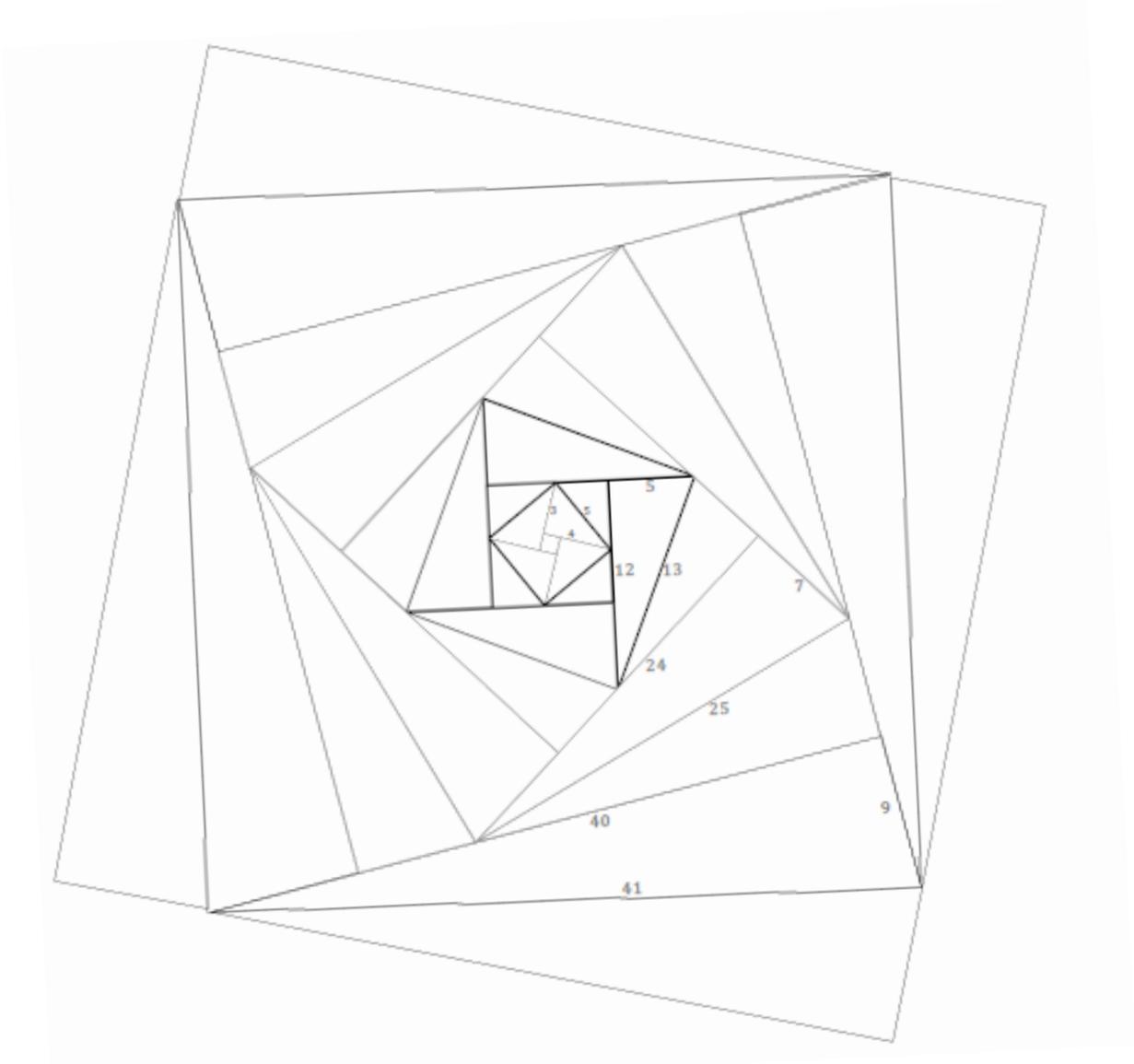
### Notes:

[BACK](#)

# Colouring Triples

*\*Credit to Yummy Math*

## Problem:



## Extension:

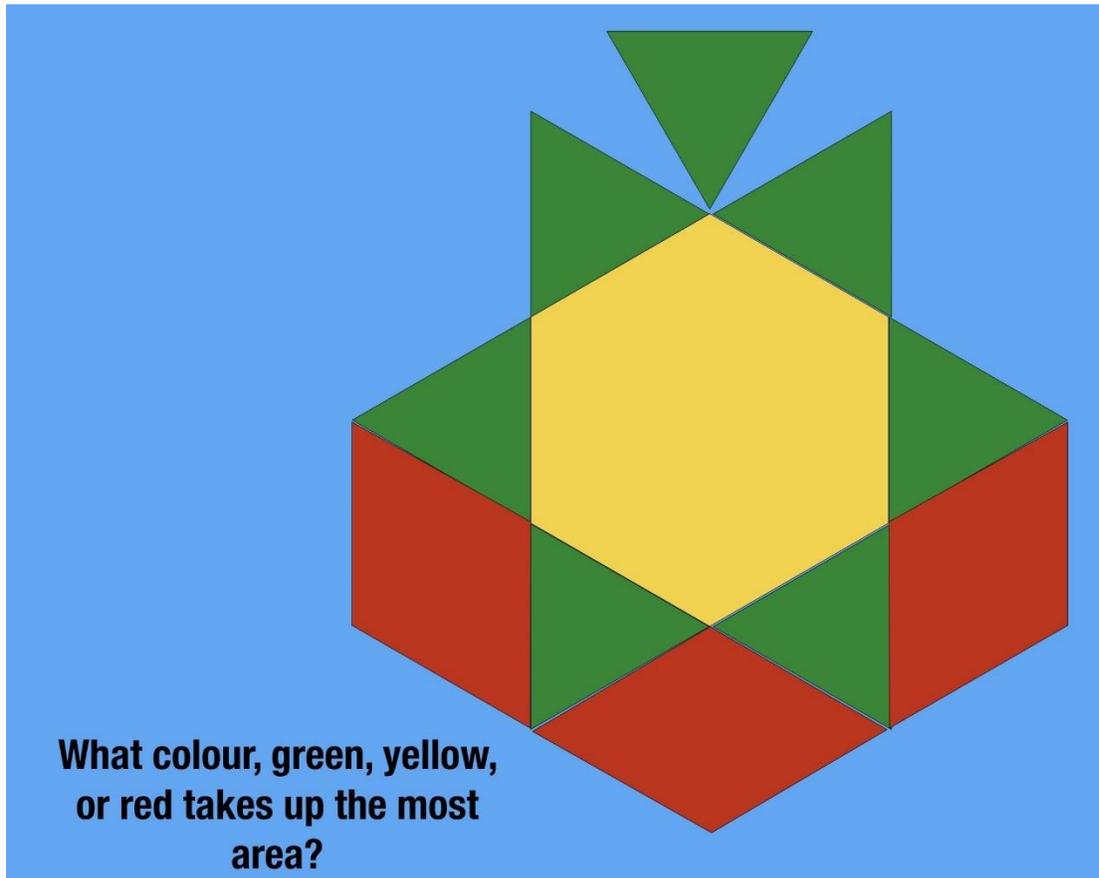
## Notes:

[BACK](#)

## Pattern Block Design

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**Problem:**



---

**Extension:**

Make your own design.

---

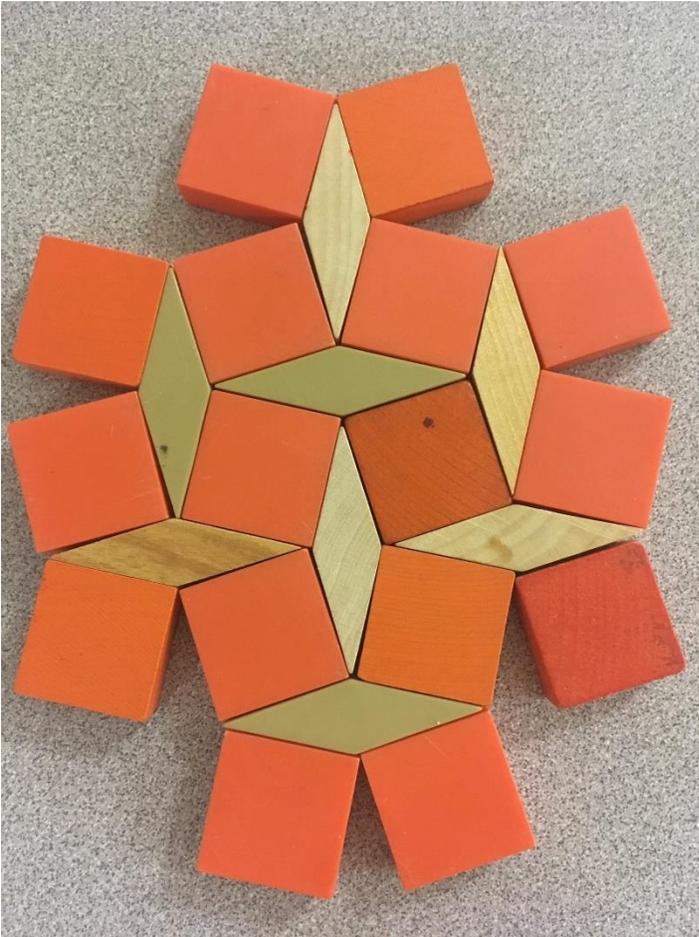
**Notes:**

[BACK](#)

## More Pattern Blocks

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**Problem:**



If you keep this pattern going, will there be more orange squares or tan rhombi?

---

**Extension:**

Make your own design.

---

**Notes:**

[BACK](#)

# Wash the Dishes

---

*\*Credit to Peter Liljedahl*

## Problem:

My brother and I have a deal every night after supper to decide who has to wash the dishes. We put two red chips and a blue chip in a bag. We draw two chips from the bag. If they are the same colour my brother washes the dishes, if they are different colours, I wash the dishes.

Is this fair?

---

## Extension:

If not, can you make it fair?

---

## Notes:

[BACK](#)

## Find the Sum

---

*\*Credit to Alan H. Schoenfeld*

### Problem:

What is the sum of the numbers

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{(n) \times (n+1)} \quad ?$$

---

### Extension:

For those of you who've seen this series, how about

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \dots + \frac{n}{(n+1)!} \quad ?$$

---

### Notes:

<https://www.researchgate.net/publication/286267296> Reflections on a course in mathematical problem solving

[BACK](#)

# Lucky Division

---

*\*Credit to Alan H. Schoenfeld*

## Problem:

Take any three-digit number and write it down twice, to make a six-digit number. (For example, the three-digit number 789 gives us the six-digit number 789 789.)

I'll bet you \$1.00 that the six-digit number you've just written down can be divided by 7, without leaving a remainder.

OK, so I was lucky. Here's a chance to make your money back, and then some.

Take the quotient that resulted from the division you just performed. I'll bet you \$5.00 that quotient can be divided by 11, without leaving a remainder.

OK, OK, so I was very lucky. Now you can clean up. I'll bet you \$25.00 that the quotient of the division by 11 can be divided by 13, without leaving a remainder?

Well, you can't win 'em all. But, you don't have to pay me if you can explain why this works.

---

## Extension:

---

## Notes:

<https://www.researchgate.net/publication/286267296> Reflections on a course in mathematical problem solving

[BACK](#)

## Solutions

---

### Problem:

For what values of "a" does the pair of equations

$$\left\{ \begin{array}{l} x^2 - y^2 = 0 \\ (x - a)^2 + y^2 = 1 \end{array} \right\}$$

have either 0, 1, 2, 3, 4, 5, 6, 7, or 8 solutions?

---

### Extension:

---

### Notes:

[https://www.researchgate.net/publication/286267296\\_Reflections\\_on\\_a\\_course\\_in\\_mathematical\\_problem\\_solving](https://www.researchgate.net/publication/286267296_Reflections_on_a_course_in_mathematical_problem_solving)

[BACK](#)

# Magic Math

---

*\*Credit to Alan H. Schoenfeld*

## Problem:

Here is a magic trick. Take any odd number, square it, and subtract 1. Take a few others and do the same thing. Notice anything? Does it always happen? Must it? Can you say why?

---

## Extension:

---

## Notes:

<https://www.researchgate.net/publication/286267296> Reflections on a course in mathematical problem solving

[BACK](#)

## Play with Percents

---

Problem:

20	30	8	15	36	10
50	9	40	45	24	60
4	1	16	5	21	12

10%	20%	30%	40%	50%	60%	70%	80%	90%
								$\frac{36}{40}$

---

Extension:

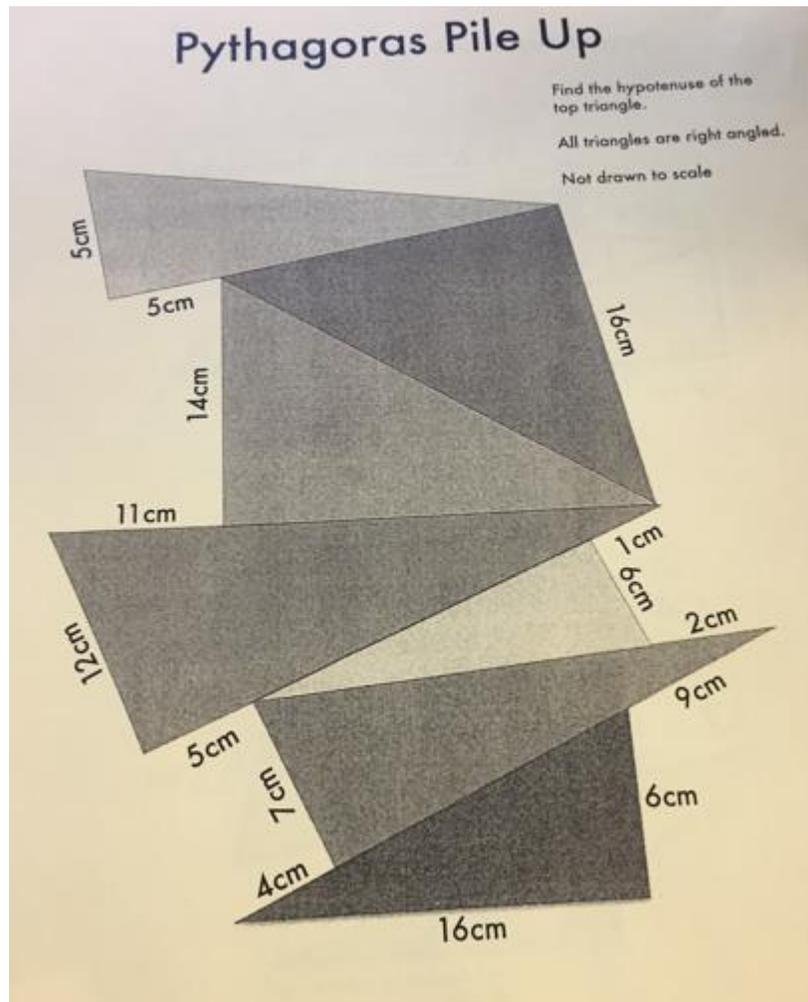
---

Notes:

# Pythagoras Pile Up

---

**Problem:**



**Extension:**

Can you make your own?

---

**Notes:**

[BACK](#)

## Sum of Cubes

---

*\*Credit to Alan H. Schoenfeld*

### Problem:

Since  $3^2 + 4^2 = 5^2$ , we know that there are three consecutive positive whole numbers with the property that the sum of the squares of the first two equals the square of the third. Can you find three consecutive positive whole numbers with the property that the sum of the *cubes* of the first two equals the *cube* of the third?

---

### Extension:

---

### Notes:

<https://www.researchgate.net/publication/286267296> Reflections on a course in mathematical problem solving

[BACK](#)

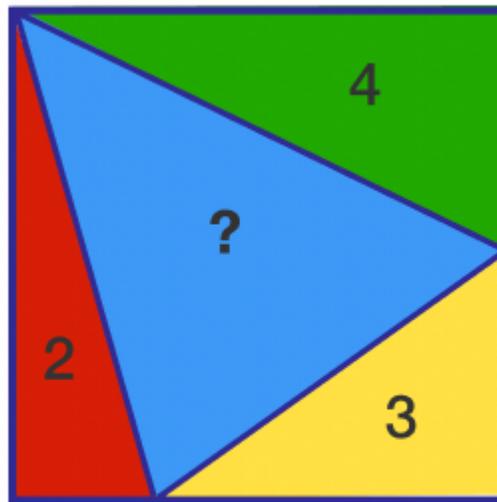
## Blue Area

---

*\*Credit to Brilliant.org*

**Problem:**

What is the **blue** area?



**Think outside the box.**

---

**Extension:**

---

**Notes:**

[BACK](#)

# Square Numbers

---

*\*Credit to Alan H. Schoenfeld*

## Problem:

Since  $3^2 + 4^2 = 5^2$ , we know that there are three consecutive positive whole numbers with the property that the sum of the squares of the first two equals the square of the third. Can you find three consecutive positive whole numbers with the property that the sum of the *cubes* of the first two equals the *cube* of the third?

---

## Extension:

---

## Notes:

<https://www.researchgate.net/publication/286267296> Reflections on a course in mathematical problem solving

[BACK](#)

## How Old is the Shepherd?

---

*\*Credit to Robert Kaplinsky*

**Problem:**

There are 125 sheep  
and 5 dogs in a flock.  
How old is the shepherd?

Solve, then watch this video <https://www.youtube.com/watch?v=kibaFBgaPx4>

---

**Extension:**

---

**Notes:**

[BACK](#)

# I Can Guess Your Number

---

*\*Credit to International Space Station You Tube*

## Problem:

- Think Of A Number
- Double It
- Add 6
- Half it
- Take Away The Original Number

What's happening? How does this work? Why?

---

## Extension:

---

## Notes:

Solve, then watch this video [https://youtu.be/h944G\\_9lnwI](https://youtu.be/h944G_9lnwI)

[BACK](#)

# I Can Guess Your Birthday

---

*\*Credit to tricks4fun You Tube*

## Problem:

<https://youtu.be/NBo99GYKL6A>

What's happening? How does this work? Why?

---

## Extension:

Investigate different "math tricks". You Tube has lots.

---

## Notes:

[BACK](#)

# Kruskal's Card Count

---

*\*Credit to Peter Liljedahl*

## Problem:

<https://youtu.be/8LG1ahXci4>

What happens? Does it always happen? Why? Can you explain it using math?

---

## Extension:

Investigate and Explore more of Peter Liljedahl's collection of card tricks.

<http://www.peterliljedahl.com/teachers/card-tricks>

Try with words on a page. Does it work?

---

## Notes:

Decks of cards.

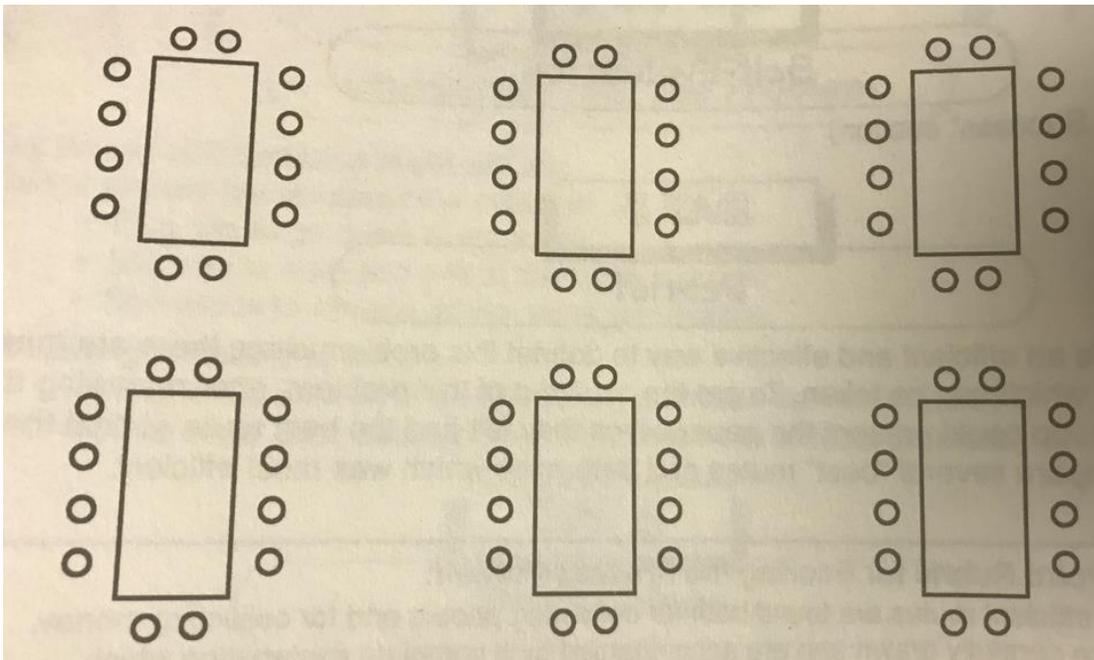
[BACK](#)

## The Paper Route

*\*Credit to Problem of the Week, Jane Fesler*

### Problem:

Elva has a paper route. She has to deliver papers to houses on six blocks. Elva needs to find the most efficient route to use when she is delivering papers. She wants to go down every street and to start and end at the same location. She would also like to retrace as few steps as possible. What would be the best route or routes for Elva to use to deliver her papers?



### Extension:

When Elva collects payments for her papers, she has to go to the front door of every house. What would be the best route or routes for Elva to use for collecting?

### Notes:

[BACK](#)

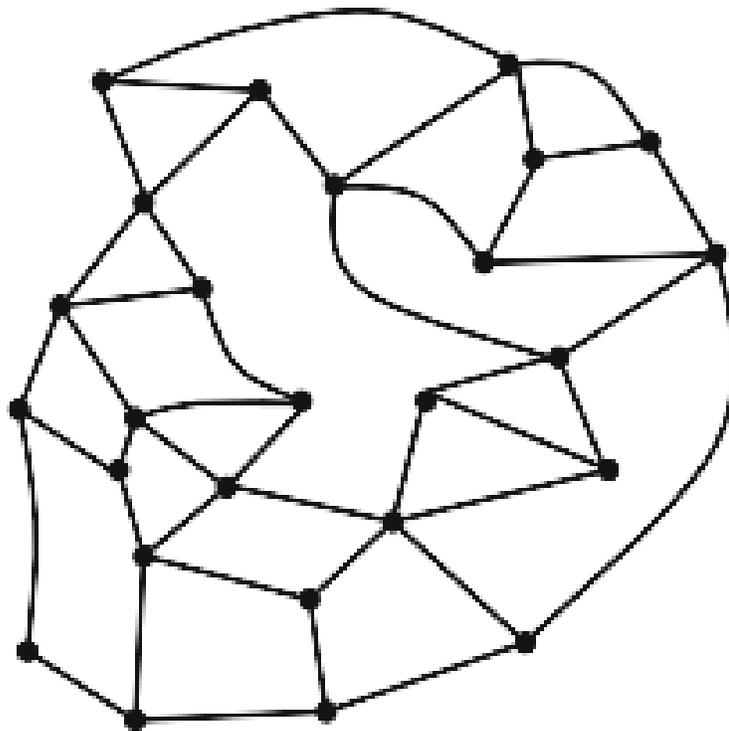
# Ice Cream Trucks

---

*\*Credit to Tim Bell*

## Problem:

Can you place ice cream trucks at intersections so that every house on every block is within a block of ice cream? Each line segment is a block. Each circle is an intersection. What is the minimum number of ice cream trucks and where should they be placed?



---

## Extension:

---

**Notes:** <http://csunplugged.org/wp-content/uploads/2014/12/unpluggedTeachersSupplement2012-draft-v1-dominatingSets.pdf>

[BACK](#)

## Divide and Conquer

---

### Problem:

I once had a teacher who could guess my number (between 1 and 100) in seven or less guesses.

Is this true?

How does he do it?

---

### Extension:

What if it was a number between 1 and 1000?

What if it was a number between 1 and 5000?

What if it was a number between 1 and  $n$ ?

---

### Notes:

[BACK](#)

# Sorting Network

*\*Credit to Tim Bell*

## Problem:

A layout like the one in Figure 1(a) is drawn on the pavement in chalk (Figure 1(b)) or on an indoor surface with painter's tape. Six students holding numbers start in the six boxes on the left, and move to the right following their respective arrow until they meet another student at a circle (node). At the circle, two students compare numbers, and the student with the smaller one follows the arrow to their left, while the student with the larger number follows the arrow to their right. Each student arrives in a new circle where they again compare numbers with the student they meet there. This structure is called a parallel sorting network because there are three comparisons happening at the same time.

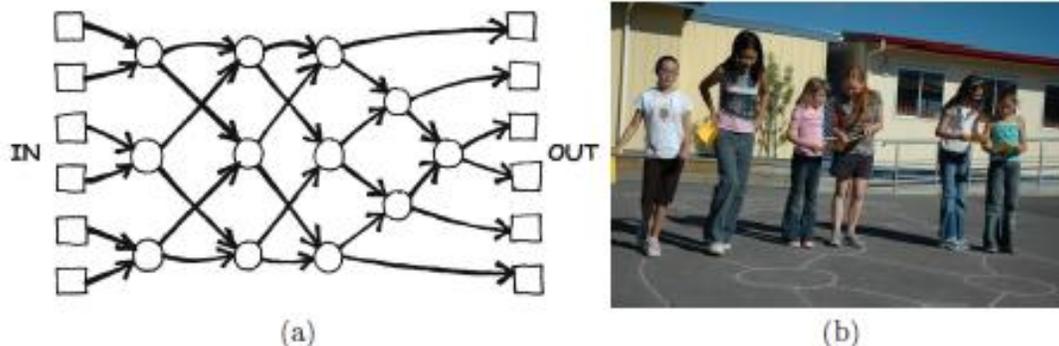


Fig. 1. (a) A 6-input parallel sorting network (b) Chalked in a school playground

## Extension:

Does it work backwards? Can you sort in descending order? Can you use it to sort 12 numbers? Or 7? Can you give it a set of numbers that will make it fail? Can you find a shorter network that still sorts the numbers? How many permutations would you have to test to check every possible input?

## Notes:

[http://csunplugged.org/wp-content/uploads/2015/03/unplugged-08-sorting\\_networks-original.pdf](http://csunplugged.org/wp-content/uploads/2015/03/unplugged-08-sorting_networks-original.pdf)

[BACK](#)

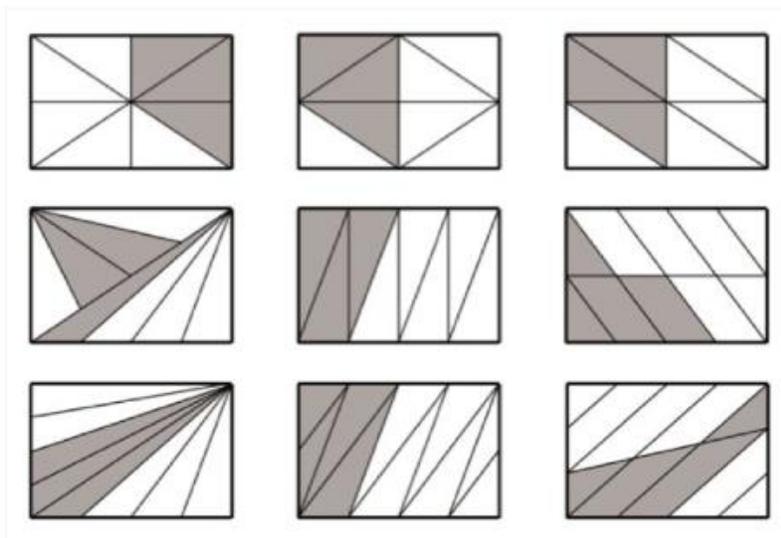
# Not Three Eighths

*\*Credit to Professor Smudge @ProfSmudge*

## Problem:

Aargh! One of these does not show  $\frac{3}{8}$ .

What does it show?



## Extension:

Can you make your own problem?

## Notes:

[BACK](#)

## Cuisenaire Red

---

### Problem:



---

### Extension:

What value are the other colour rods?  
Can you make your own problem?

---

### Notes

[BACK](#)

# Folding Halves

---

*\*Credit to Mark Chubb @MarkChubb3*

## Problem:

How many ways can you fold square sticky notes into shapes exactly  $\frac{1}{2}$  the area?

---

## Extension:

What if it wasn't  $\frac{1}{2}$ ?

---

## Notes:

[BACK](#)

## Finding Equivalent Fractions

---

*\*Credit to Fletcher, Graham, and Bowen Kerins, Open Middle*

### Problem:

Using the whole numbers 0 through 9 no more than once, create 3 equivalent fractions.

$$\frac{\square}{\square} = \frac{\square}{\square \square} = \frac{\square}{\square}$$

---

### Extension:

---

### Notes:

## Pick a Formula

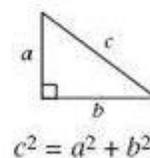
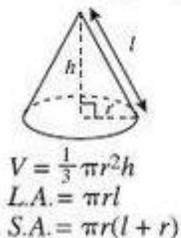
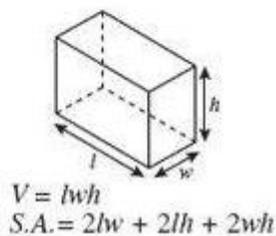
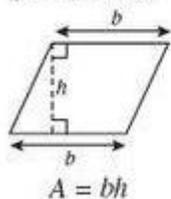
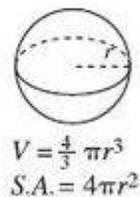
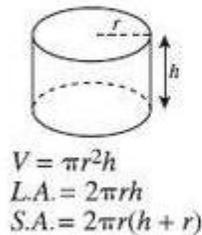
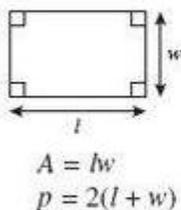
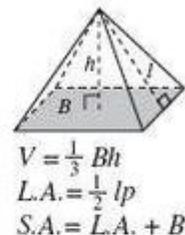
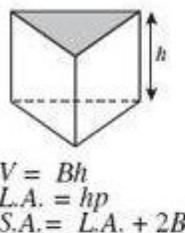
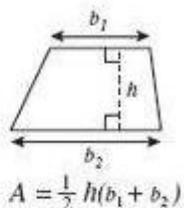
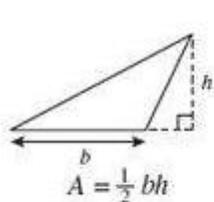
---

### Problem:

Pick a formula and explain how it works! Do you understand it? Why is it written how it is written? Can you explain it better if you write it a different way? Can you calculate area and volume without formulas? Investigate and explain!

### Geometry Formula Sheet

#### Geometric Formulas



### Extension:

---

### Notes:

[BACK](#)

# Prime Climb

*\*Credit to Prime Climb Game, Math 4 Love*

## Problem:



What do you notice? What do you wonder? What do you want to investigate?

## Extension:

## Notes:

[BACK](#)

## In and Out

---

*\*Credit to Open Middle*

### Problem:

<b>Open Middle</b>	
<ul style="list-style-type: none"><li>• Use the digits 0–9 to complete the function table</li><li>• Each digit can only be used once</li></ul>	
IN	OUT
<input type="text"/>	<input type="text"/>
<input type="text"/> <input type="text"/>	<input type="text"/> <input type="text"/>
<input type="text"/> <input type="text"/>	<input type="text"/> <input type="text"/>

---

### Extension:

Can you create your own rule and your own table and get someone to solve it?

---

### Notes:

[BACK](#)

## Two Equations

*\*Credit to Robert Kaplinsky at [www.robertkaplinsky.com](http://www.robertkaplinsky.com)*

### Problem:

Using the digits 1 to 9, at most one time each, create two equations: one where  $x$  has a positive value and one where  $x$  has a negative value.

$$\boxed{\phantom{00}} + x = \boxed{\phantom{00}}$$

---

### Extension:

What other equations can you make with integers?

---

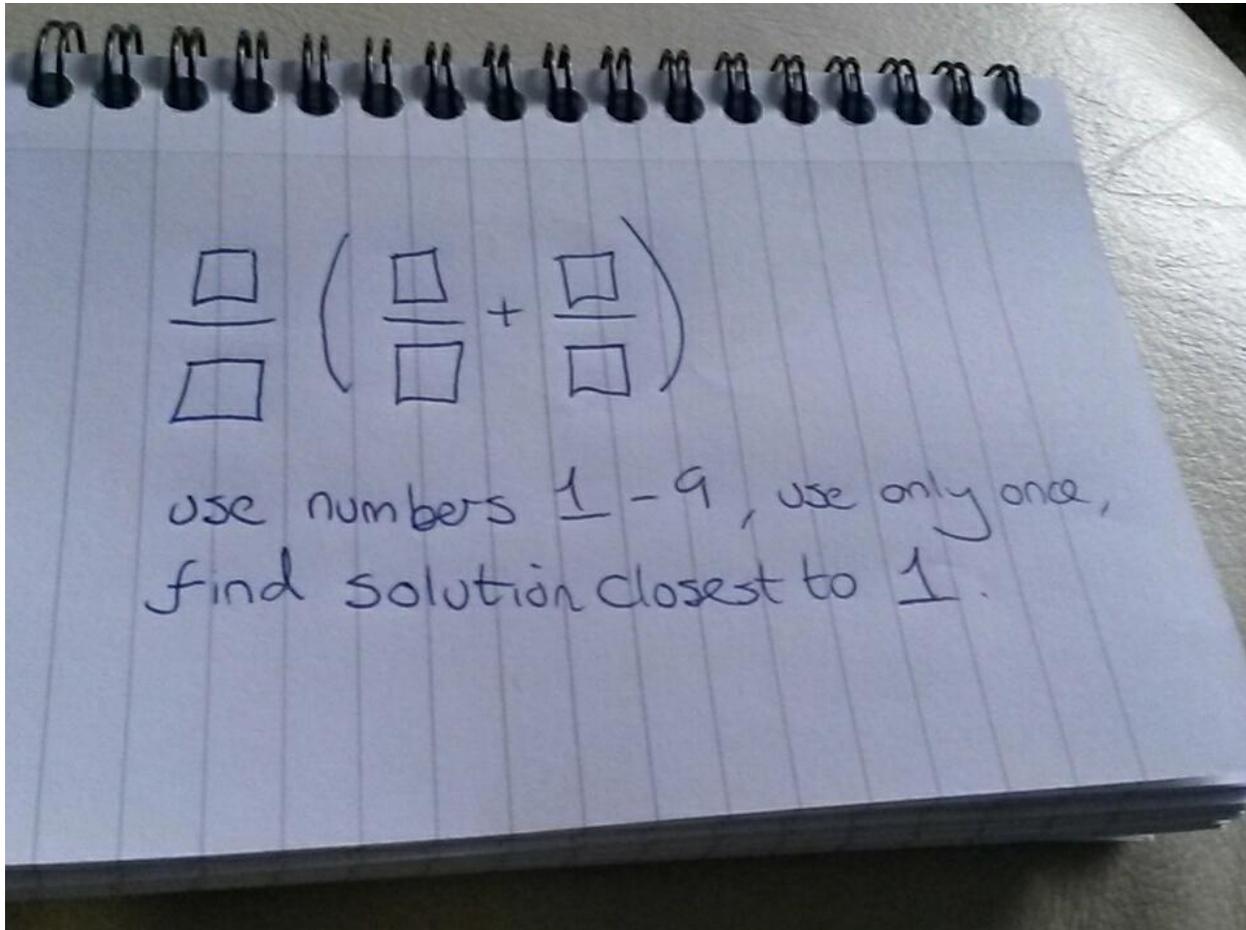
### Notes:

[BACK](#)

## Closest to One

*\*Credit to Robert Kaplinsky at [www.robertkaplinsky.com](http://www.robertkaplinsky.com)*

### Problem:



### Extension:

What if it was furthest to one?

What if it wasn't one?

### Notes:

[BACK](#)

## Square Moat

---

### Problem:

You are trying to rescue your friend from a castle. The castle is surrounded by a square moat with a uniform width of 20 feet. You have two planks that measure 19 feet each. How do you get across the moat to the castle?

---

### Extension:

What is the minimum feet of plank you need?

---

### Notes:

# Math Exploration

---

*\*Credit to Steven Strogatz*

## Problem:

Today in math explorations class: what whole numbers can you make by adding 3s and 5s? Prove your conjecture!

---

## Extension:

---

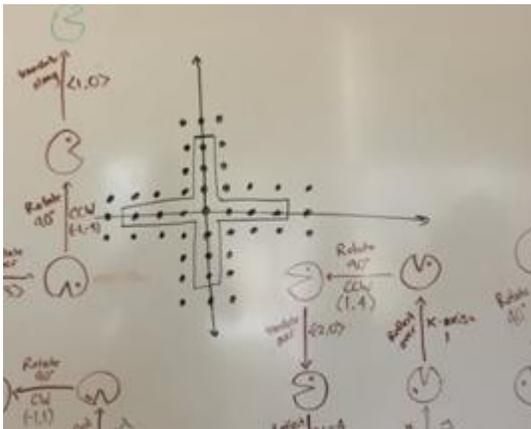
## Notes:

[BACK](#)

# Pac Man Transformations

---

## Problem:



How many moves does it take Pac Man to eat the dots around this shape? What moves does he have to make? Be specific with your transformations.

---

## Extension:

What if it was a different shape?

---

## Notes:

## Abc Denominators

---

*\*Credit to NCTM, Mathematics Teacher*

### Problem:

For which positive integers  $a, b, c$  will  
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1?$$

Can you find them all?

---

### Extension:

---

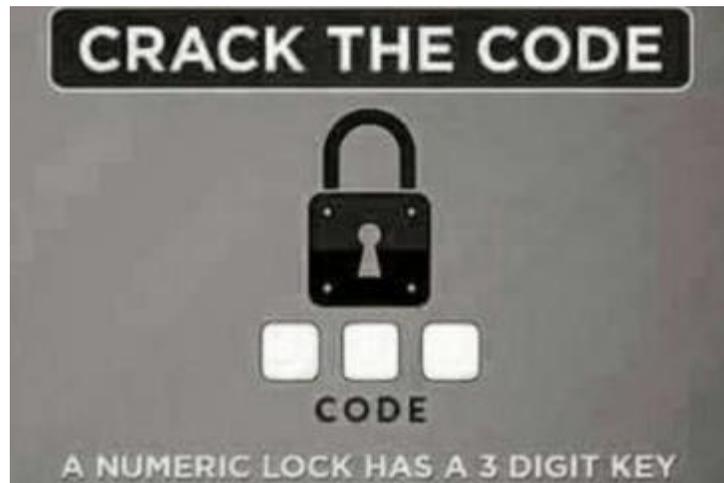
### Notes:

[BACK](#)

## Crack the Code

---

### Problem:



### Hints:

- |       |  |
|-------|--|
| 6 8 2 | One digit is correct and correctly placed.     |
| 6 1 4 | One digit is correct but incorrectly placed.   |
| 2 0 6 | Two digits are correct but incorrectly placed. |
| 7 3 8 | Nothing is correct                             |
| 7 8 0 | One digit is correct but incorrectly placed.   |

---

### Extension:

---

### Notes:

[BACK](#)

## Square Root It

---

*\*Credit to Manan Shah at mathmisery.com*

### Problem:

Use only the digits 0 to 9. No leading zeros. Each letter maps to one and only one number and vice versa.

$$\sqrt{\text{BELIEFS}} = \text{MATH}$$

---

### Extension:

---

### Notes:

[BACK](#)

# Repeat Offender

---

*\*Credit to Mr. Enlow*

## Problem:

Is 11 the only prime number whose digits are all the same?

---

## Extension:

---

## Notes:

[BACK](#)

# Counting Divisors

---

*\*Credit to Mr. Enlow*

## Problem:

How many positive divisors does 201 have?

---

## Extension:

---

## Notes:

[BACK](#)

# Strange Powers

---

*\*Credit to Mr. Enlow*

## Problem:

What proportion of the powers of 2 start with a one?

---

## Extension:

---

## Notes:

[BACK](#)

# Rectangles into Squares

---

*\*Credit to Mr. Enlow*

## Problem:

What is the smallest number of squares into which a 22 by 36 rectangle can be divided?

---

## Extension:

---

## Notes:

[BACK](#)

# Squerritories

---

*\*Credit to Mr. Enlow*

## Problem:

Imagine the coordinate plane divided into 1-unit squares, whose vertices are the points with integer coordinates. If a line segment is drawn from  $(0,0)$  to  $(34, 21)$ , how many of those 1-unit squares does it enter?

---

## Extension:

What if the coordinates are different?

---

## Notes:

[BACK](#)

# My Outsides Match my Insides

---

*\*Credit to Mr. Enlow*

## Problem:

You have an infinite supply of square tiles, some black and some white. You can make closed shapes with these tiles, with the white tiles forming the interior, and the black tiles forming the border. How many shapes can you find that use equal numbers of white and black tiles?

---

## Extension:

What if the coordinates are different?

---

## Notes:

[BACK](#)

# Kissing Circles

---

*\*Credit to Mr. Enlow*

## Problem:

A circle of radius 1 has inside of it two circles of radius  $\frac{1}{2}$ , so that each circle is tangent to the other two. A fourth circle is drawn so that it is tangent to each of the first three. Find the radius of the fourth circle.

---

## Extension:

What if the coordinates are different?

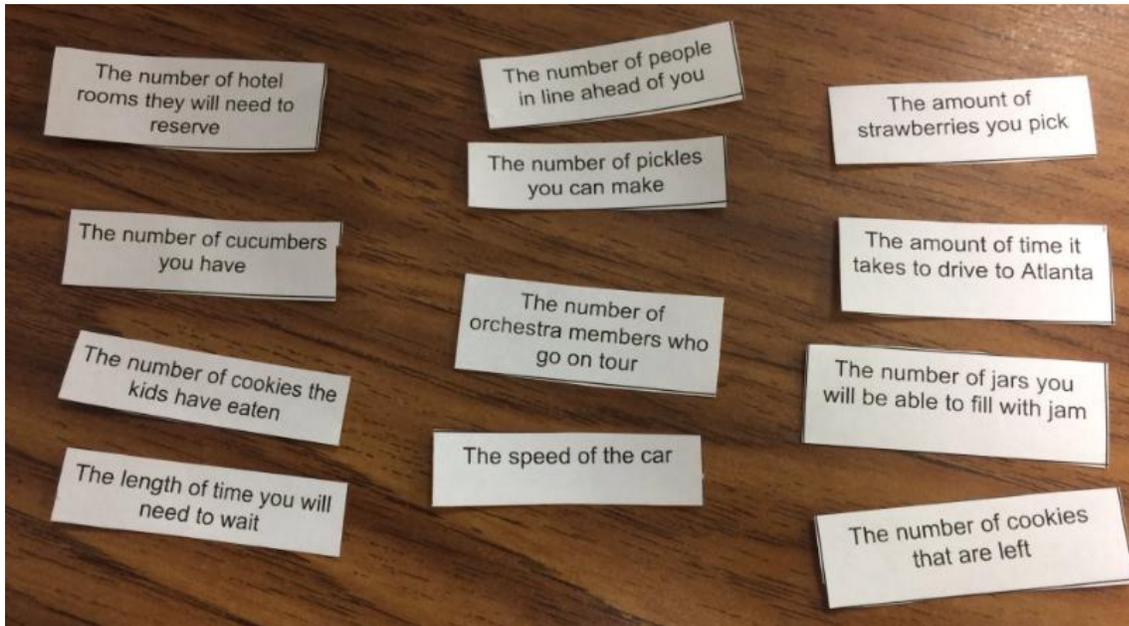
---

## Notes:

## One vs. the Other

---

### Problem:



Can you make pairs of these statements into graphs? What is on the x-axis? What is on the y-axis? How do you think the line behaves?

---

### Extension:

---

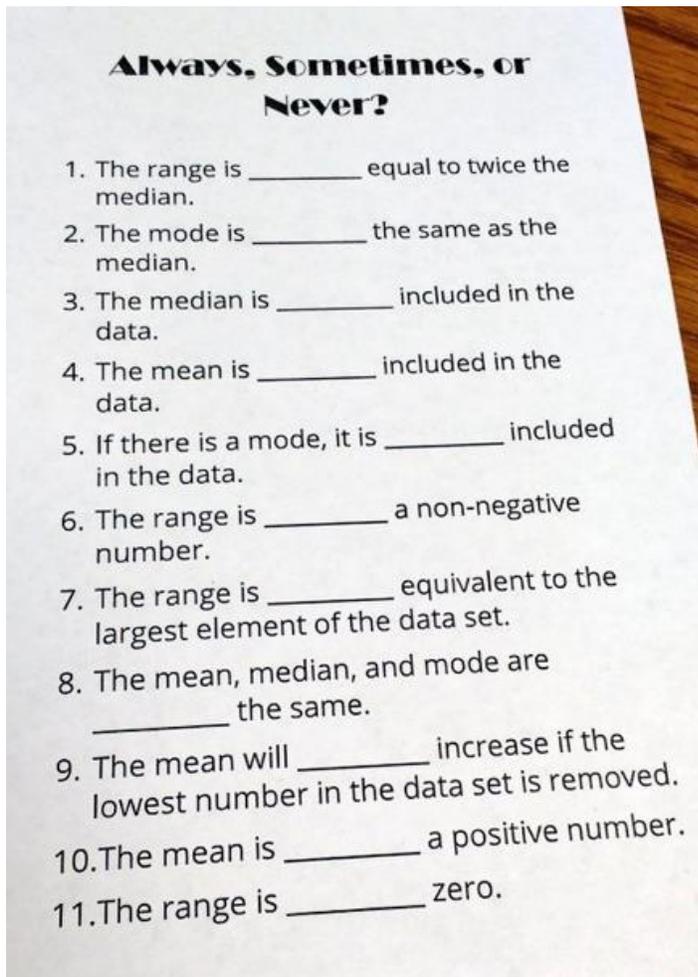
### Notes:

[BACK](#)

## Always, Sometimes, or Never

---

### Problem:



Explain and provide examples!

---

### Extension:

---

### Notes:

[BACK](#)

# Open Middle Fractions 1

---

\*Credit to <http://www.openmiddle.com/convertng-between-fractions-and-decimals/>

## Problem:

Directions: Using the numbers 0 through 9, at most one time each, fill in each of the boxes so that the fraction equals the decimal.

$$\frac{\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array}}{\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array}} = \square . \square \square$$

---

## Extension:

---

## Notes:

[www.openmiddle.com](http://www.openmiddle.com)

[BACK](#)

## Open Middle Fractions 2

---

\*Credit to <http://www.openmiddle.com/fractions-less-than-one-half/>

### Problem:

Directions: Using the whole numbers 1 through 9 as numerators or denominators, how many fractions can you make that are less than one half?

$$\frac{\square}{\square} < \frac{1}{2}$$

---

### Extension:

---

### Notes:

[www.openmiddle.com](http://www.openmiddle.com)

[BACK](#)

## Open Middle Fractions 3

---

\*Credit to <http://www.openmiddle.com/adding-fractions-4/>

### Problem:

Directions: Using the whole numbers 1 through 10, at most one time each, fill in the boxes so that the sum is equal to 1 whole.

$$\frac{\boxed{\phantom{0}}}{\boxed{\phantom{0}}} + \frac{\boxed{\phantom{0}}}{\boxed{\phantom{0}}} + \frac{\boxed{\phantom{0}}}{\boxed{\phantom{0}}} = 1$$

---

### Extension:

---

### Notes:

[www.openmiddle.com](http://www.openmiddle.com)

[BACK](#)

## Open Middle Fractions 4

---

\*Credit to <http://www.openmiddle.com/adding-mixed-numbers-3/>

### Problem:

Directions: Use the digits 1-9 each once to make a the largest possible sum.

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array}$$

---

### Extension:

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### Notes:

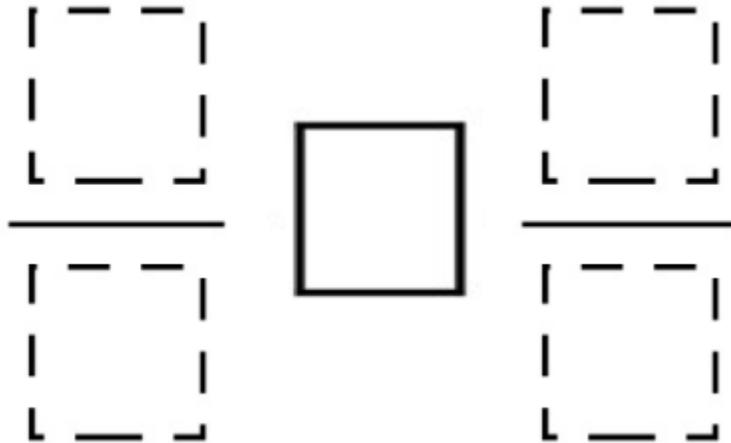
[www.openmiddle.com](http://www.openmiddle.com)

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## Open Middle Fractions 5

---

**Problem:**



Place digits 2-9 once at the most and one operation to make the smallest possible result.

---

**Extension:**

---

**Notes:**

[www.openmiddle.com](http://www.openmiddle.com)

[BACK](#)

## Same Perimeter

---

### Problem:

All these shapes have the same perimeter, except one.  
Which shape is the odd one out?

A B C D E F

---

### Extension:

---

### Notes:

# Cheryl's Birthday

---

*\*Credit to the internet*

## Problem:

<https://www.nytimes.com/2015/04/15/science/a-math-problem-from-singapore-goes-viral-when-is-cheryls-birthday.html?mcubz=1>

Albert and Bernard just met Cheryl. "When's your birthday?" Albert asked Cheryl. Cheryl thought a second and said, "I'm not going to tell you, but I'll give you some clues." She wrote down a list of 10 dates:

May 15, May 16, May 19

June 17, June 18

July 14, July 16

August 14, August 15, August 17

"My birthday is one of these," she said.

Then Cheryl whispered in Albert's ear the month — and only the month — of her birthday. To Bernard, she whispered the day, and only the day.

"Can you figure it out now?" she asked Albert.

Albert: I don't know when your birthday is, but I know Bernard doesn't know, either.

Bernard: I didn't know originally, but now I do.

Albert: Well, now I know, too!

When is Cheryl's birthday?

---

## Extension:

---

## Notes:

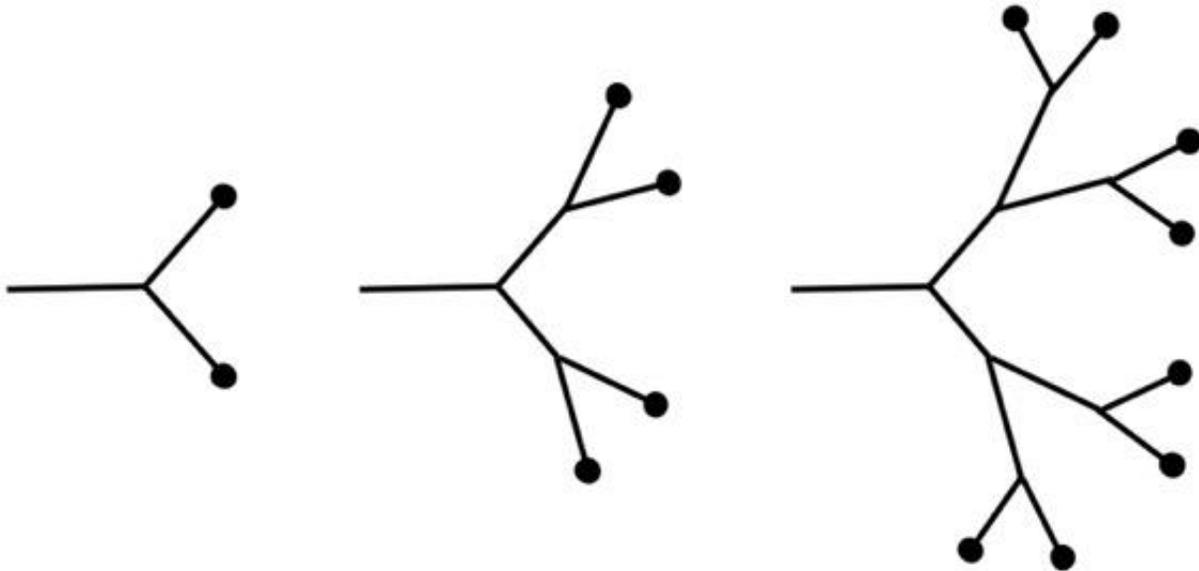
<https://www.youtube.com/watch?v=emiMj8cCL5E>

[BACK](#)

## Picture Pattern

---

**Problem:**



What do you notice? What do you wonder?

---

**Extension:**

---

**Notes:**

<https://www.youtube.com/watch?v=emiMj8cCL5E>

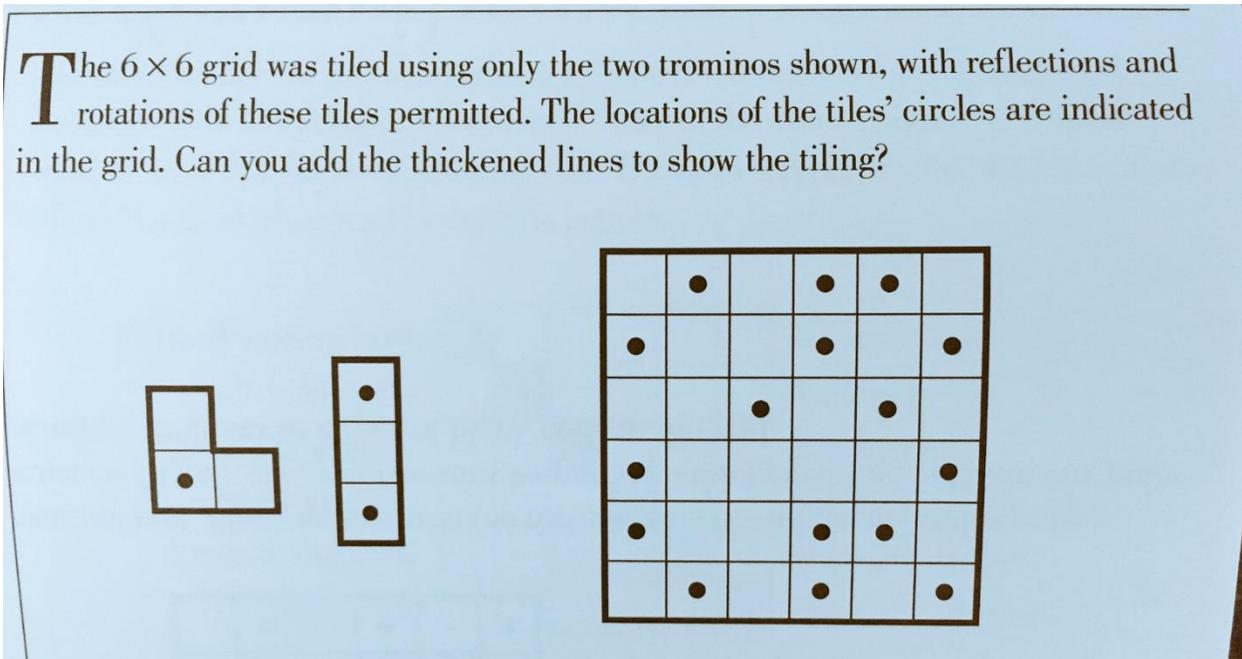
[BACK](#)

## Tiling Trominos

---

### Problem:

The  $6 \times 6$  grid was tiled using only the two trominos shown, with reflections and rotations of these tiles permitted. The locations of the tiles' circles are indicated in the grid. Can you add the thickened lines to show the tiling?




---

### Extension:

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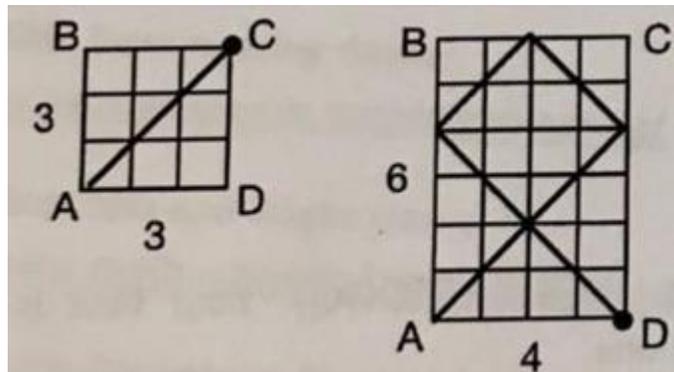
### Notes:

[BACK](#)

# Billiards

\*Credit to Jamie Mulholland and Richard Hoshino

## Problem:



You hit a ball at a 45 degree angle from the lower left corner, A, of a rectangular billiards table. The ball rebounds off each side in a new direction, but at the same angle. As the dimensions of the billiards table varies, explore which of the four corner pockets the ball can end up in.

## Extension:

Can you predict which corner the ball will end up for the table with dimensions 108 by 94?

For a general  $n \times m$  table, how many times does a ball hit an edge before going into a corner hole?

For a general  $n \times m$  table, how many  $1 \times 1$  squares on graph paper does the path traverse before going into a corner hole?

## Notes:

Graph paper. Start with tables of smaller dimensions.

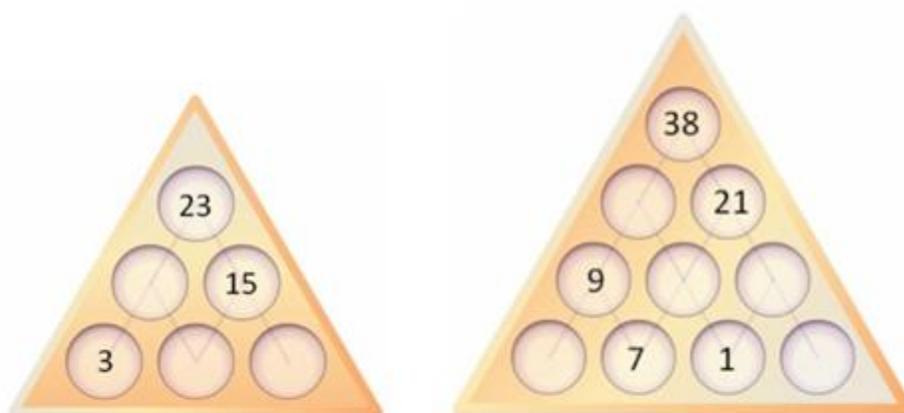
[BACK](#)

# Triangle Pattern Puzzles

\*Credit to <https://nrich.maths.org/2281>

## Problem:

To complete these puzzles, the sum of two horizontally adjacent cells equals the cell above.



What is the minimum number of clues necessary for any puzzle to have a unique solution? Does it matter where these clues are located?

## Extension:

What about a 3-D version of this puzzle?

## Notes:

[BACK](#)

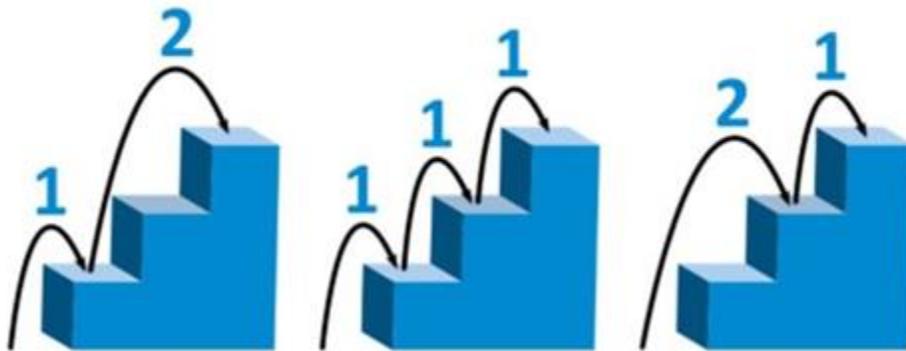
# Climbing Steps

---

\*Credit to <https://www.playwithyourmath.com>

## Problem:

If you can only climb one step or two steps at a time, in how many ways can you climb 3 steps? 4 Steps? 10 steps? 15 steps?  $n$  steps?



---

## Extension:

What do you notice and why?

What if you could also climb three steps at a time?

---

## Notes:

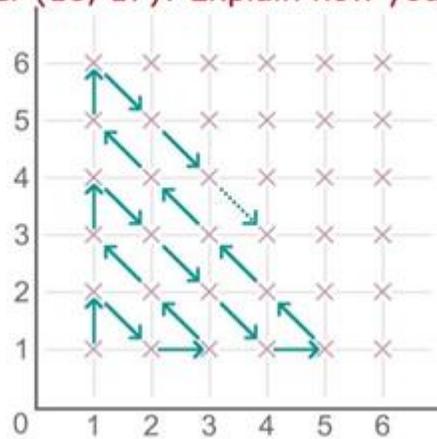
[BACK](#)

# Route to Infinity

\*Credit to <https://nrich.maths.org/5469>

## Problem:

If the pattern of arrows continues for ever, which point will the route visit immediately after  $(18, 17)$ ? Explain how you know.



## Extension:

How many points will be visited before the route reaches the point  $(9,4)$ ?

Which point will be the 1000<sup>th</sup> to be visited?

## Notes:

[BACK](#)

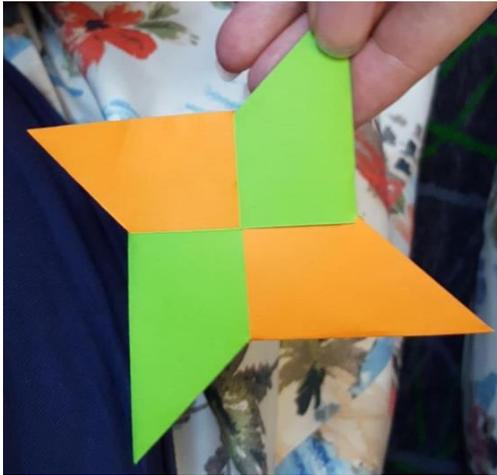
# Paper Stars

---

**Problem:**

[https://www.youtube.com/watch?v=AyOID1\\_xm4w](https://www.youtube.com/watch?v=AyOID1_xm4w)

Watch this video and make a paper star.



---

**Extension:**

Can you add your own words to the video to explain to someone how to make a paper star?

---

**Notes:**

Lots of paper folding tutorials on youtube. Explore!

[BACK](#)

## Family of Related Sequences

---

**Problem:**

Here are the first few sequences from a family of related sequences:

$$A_0 = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, \dots$$

$$A_1 = 2, 6, 10, 14, 18, 22, 26, 30, 34, \dots$$

$$A_2 = 4, 12, 20, 28, 36, 44, 52, \dots$$

$$A_3 = 8, 24, 40, 56, 72, \dots$$

$$A_4 = 16, 48, 80, \dots$$

$$A_5 = 32, 96, \dots$$

$$A_6 = 64, \dots$$

What do you notice? What do you wonder?

---

**Extension:**

---

**Notes:**

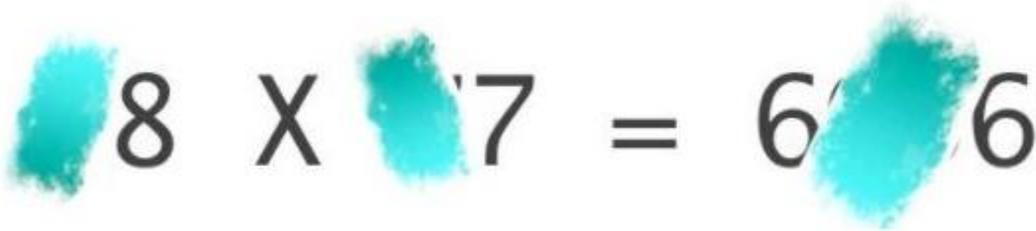
[BACK](#)

## Three Smudges

---

*\*Credit to Peter Liljedahl's Smudged Math*

**Problem:**


$$\text{Smudge} 8 \times \text{Smudge} 7 = 6 \text{Smudge} 6$$

How many different ways can you solve this?

---

**Extension:**

---

**Notes:**

[BACK](#)

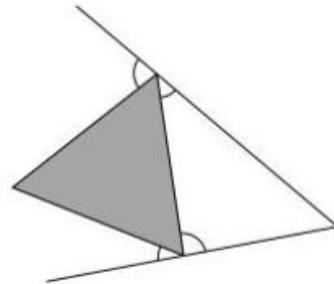
# Four Angles

---

## Problem:

The diagram shows an equilateral triangle touching two straight lines.

What is the sum of the four marked angles?



---

## Extension:

---

## Notes:

## The Professor's Toads

---

*\*Credit to @scottmckenzie27*

### Problem:

**A professor is buying toads and frogs at the university. He is paying 3¢ for each toad, and 5¢ for each frog. Two boys brought him a bag with 10 animals inside. He gave them 38¢. How many were toads and how many were frogs?**

How many different ways can you solve this problem?

---

### Extension:

What if it wasn't 10 animals?

---

### Notes:

[BACK](#)

## Twitter Litter

---

### Problem:

Ben's Twitter account has 500 followers. When he sends a tweet,  $\frac{1}{4}$  of his followers retweet his message, and 40% just read it.

**Among his remaining followers**, 15 out of every 25 just ignore his chatter, while the rest choose to stop following him completely.

When Ben sends this tweet, how many of his original 500 followers:

- a) retweet it?
- b) read it?
- c) ignore it?
- d) stop following him?



---

### Extension:

---

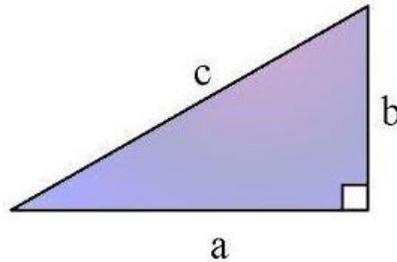
### Notes:

## Length of the Hypotenuse

---

Problem:

**Starting with 5, every second Fibonacci number is the length of the hypotenuse of a right triangle with integer sides.**



Is this true? Can you prove or disprove it?

---

Extension:

---

Notes:

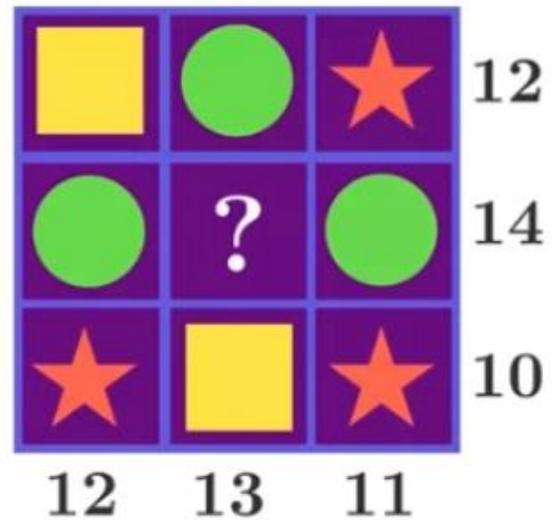
[BACK](#)

## Middle Square

---

Problem:

Which of the three shapes goes in the center square?



---

Extension:

Can you make your own puzzle?

---

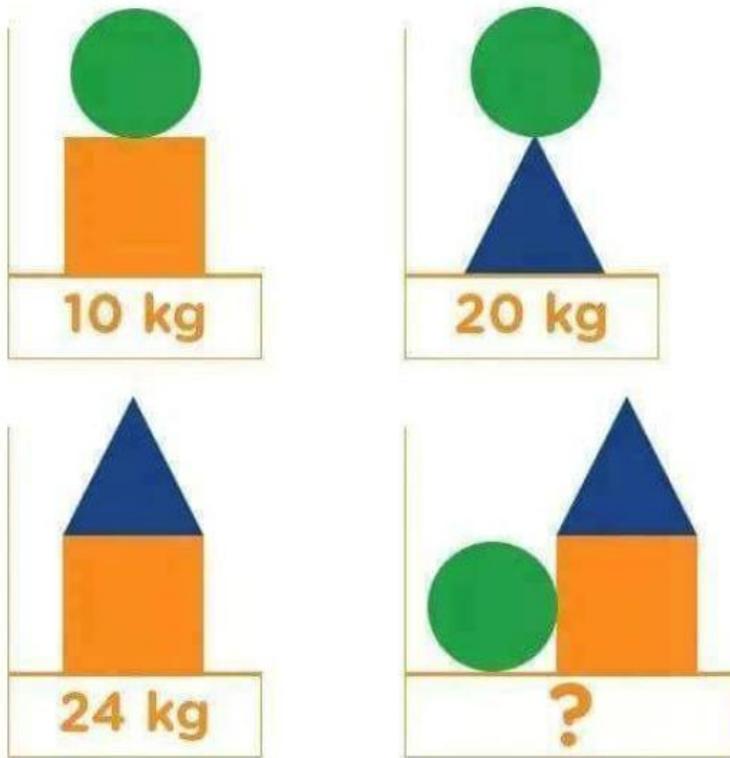
Notes:

[BACK](#)

## Missing Weight

---

Problem:



---

Extension:

Can you make your own puzzle?

---

Notes:

[BACK](#)

# Conjectures

---

*\*Credit to some unsolved problems*

## Problem:

Research and explore an unsolved math problem that you are interested in. Here are some examples.

[Beal Conjecture](#)

[Twin Prime Conjecture](#)

[Fermat's Last Theorem](#)

[Goldbach Conjecture](#)

---

## Extension:

---

## Notes:

Here are some more unsolved problems

[http://unsolvedproblems.org/index\\_files/Beal.htm](http://unsolvedproblems.org/index_files/Beal.htm)

[BACK](#)

## Misunderstood Area and Volume

---

*\*Credit to all of my mathematical misunderstandings*

### Problem:

Think about a 2 by 2 by 2 cube.

Now think of its volume. 8 cubic units right?

Now think about a 0.2 by 0.2 by 0.2 cube.

Think of its volume – 0.008 cubic units!

What about the area of a square tile that measures  $\frac{1}{2}$  foot by  $\frac{1}{2}$  foot?

Does any of this make sense? Can you make sense of it?

Explore perimeter, area, volume relationships with fractional measurements.

---

### Extension:

What about a triangle with fractional dimensions, a pyramid?

What about a rectangular prism?

What if it was a different 2-D or 3-D shape?

---

### Notes:

[BACK](#)

# Smudged

*\*Credit to Peter Liljedahl and Smudged Math*

**Problem:**

$$1. \quad \blacksquare \div \blacksquare = 6 R4$$

$$2. \quad \blacksquare \div 5 = \blacksquare R4$$

$$3. \quad \blacksquare \div 5 = 6 R \blacksquare$$

$$4. \quad 34 \div \blacksquare = \blacksquare R4$$

$$5. \quad 34 \div \blacksquare = 6 R \blacksquare$$

$$6. \quad 34 \div 5 = \blacksquare R \blacksquare$$

What do you want to do? Do it and explain why!

---

**Extension:**

Can you make your own Smudged Math problem?

---

**Notes:**

[BACK](#)

# Nines

---

## Problem:

What happens with remainders when you divide by 9?

---

## Extension:

What if it was a different number?

---

## Notes:

[BACK](#)

# Nines Again

---

*\*Credit to Manan Shah @shahlock*

## Problem:

Math Prompt:

$$9^1 = 9 \text{ [1 digit]}$$

$$9^2 = 81 \text{ [2 digits]}$$

$$9^3 = 729 \text{ [3 digits]}$$

$$9^4 = 6561 \text{ [4 digits]}$$

---

## Extension:

Does  $9^k$  have  $k$  digits if  $k$  is an integer bigger than one?

---

## Notes:

[BACK](#)

## Find the Cards on the Table

---

*\*Credit to Ian Stewart*

### Problem:

A number of playing cards, all diamonds from one pack, were arranged in a circle on the table in such a way that the total face values of any set of three adjacent cards differed by at most one from the total for any similar set. The highest and lowest cards on the table were the 10 and 2 of diamonds. The 5 and 6 of diamonds were also in the circle.

Which other cards were on the table and what was their order?

---

### Extension:

---

### Notes:

[BACK](#)

# Fifteen

---

*\*Credit to 29e Championnat International des Jeux Mathematiques, CRSNG*

## Problem:

Can you find all of the 3 digit numbers that are divisible by 15 and whose digits add to 15? For example, the number 825.

---

## Extension:

What if it wasn't only 3 digit numbers?

---

## Notes:

[BACK](#)

# Cards

---

*\*Credit to kainetikmath*

## Problem:

Allan, Bala, and Chee had some cards. The ratio of the number of Allan's cards to the number of Bala's cards was 3:5. After Bala and Chee had both lost  $\frac{1}{2}$  of their cards, Bala had 75 more cards than Cee. The three children had 341 cards left in the end.

What do you want to know?

---

## Extension:

How many cards did Chee start with?

---

## Notes:

[BACK](#)

# Sticky Numbers

---

*\*Credit to [nrich.maths.org/roadshow](http://nrich.maths.org/roadshow)*

## Problem:

Look at the following numbers:

10   15   21   4   5

They are arranged so that each pair of adjacent numbers adds up to a square number.

Try to arrange the numbers 1 to 17 in a row in the same way, so that each adjacent pair adds up to a square number.

---

## Extension:

Can you arrange them in more than one way?

---

## Notes:

[BACK](#)

# Knots in a Rope

---

## Problem:

What happens to the length of a rope when you tie knots in it?



---

## Extension:

What if the rope was thicker?

---

## Notes:

Lengths of rope of different thickness.

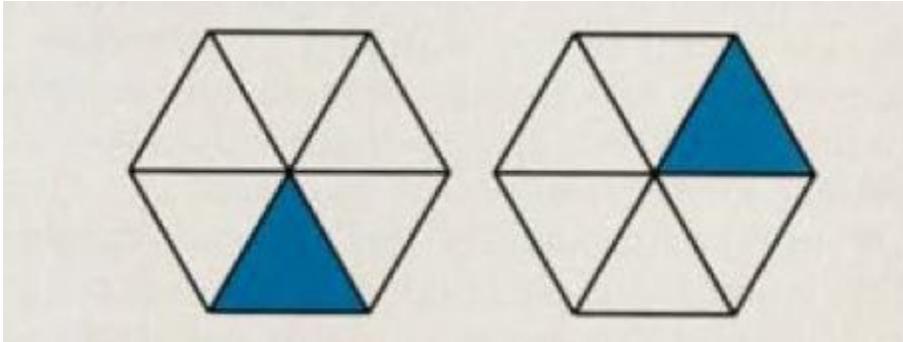
Rulers.

[BACK](#)

## Two Twelfths

---

**Problem:**



Prove mathematically that this picture represents two twelfths.

What else could it represent?

---

**Extension:**

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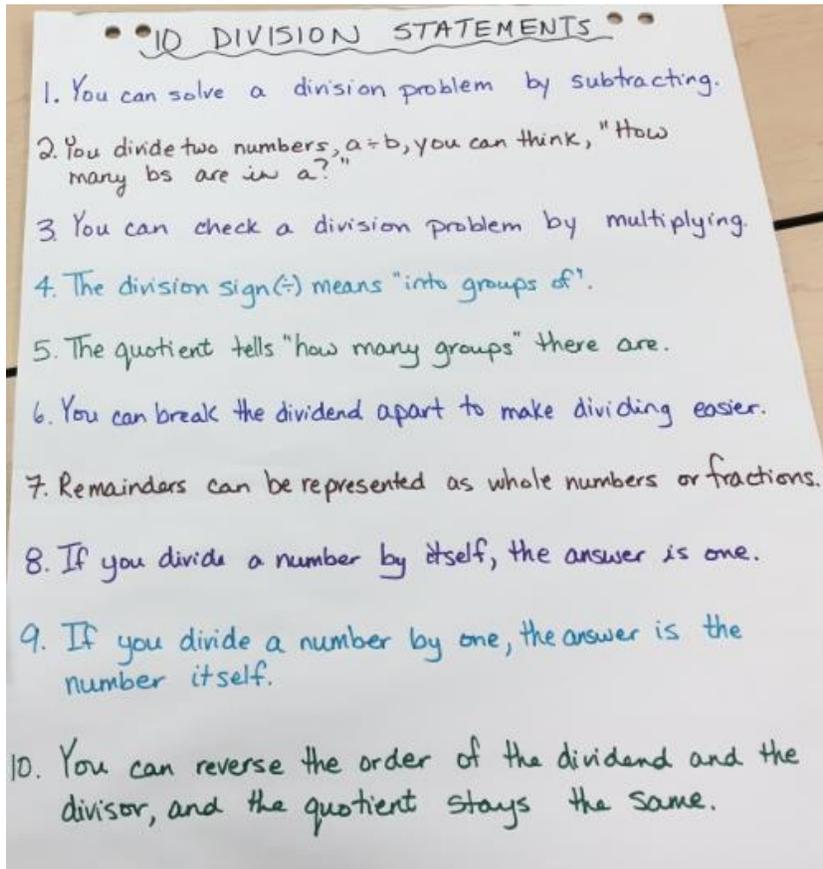
**Notes:**

# Division Statements

---

\*Credit to Marilyn Burns

## Problem:



Pick a statement. Is it true? Is it helpful?

---

## Extension:

Do you have a division statement that is more helpful?

---

## Notes:

[BACK](#)

# 1234

---

**Problem:**

You have 4 numbers, all of which include the digits 1, 2, 3, and 4. How can you arrange the 4 digits in the 4 numbers to make them add up to a sum of 9000?

---

**Extension:**

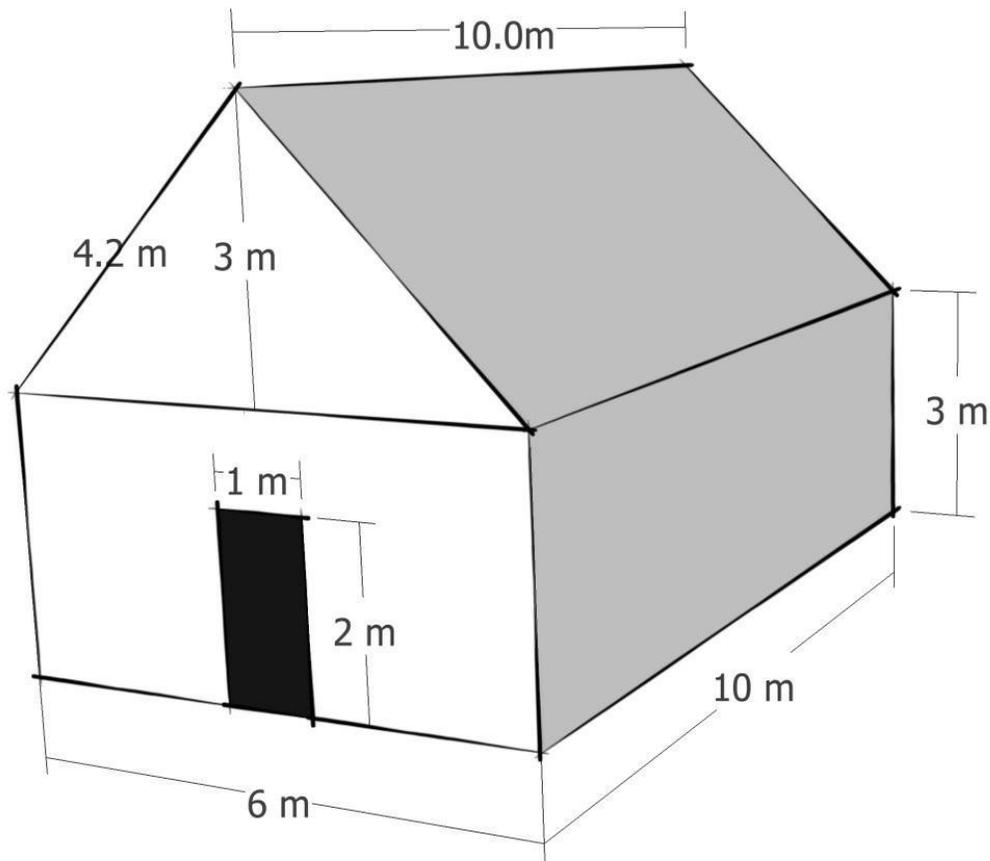
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**Notes:**

## Favourite Surface Area

*\*Credit to Nat Banting*

### Problem:



Design an expansion for this house that doubles its surface area. The expansion must share some portion of a wall with the original house.

### Extension:

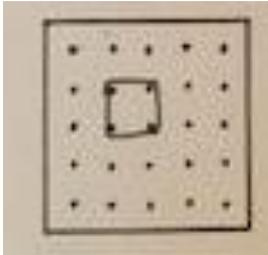
**Notes:** <http://musingmathematically.blogspot.ca/search?q=house>

[BACK](#)

## Pick's Theorem

---

### Problem:



Pick's Theorem allows you to find the area of any shape on the geoboard from the number of pegs on the perimeter of the shape and the number of pegs inside the shape. The above square with an area of one has four pegs on its perimeter and zero pegs inside.

Investigate other shapes with four pegs on the perimeter and zero pegs inside. Compare their areas.

What about shapes with four pegs on the perimeter and one peg inside? Two pegs inside? Three? Four? Five?

---

### Extension:

Can you find a master formula that allows you to figure out the area for any combination of pegs on the perimeter and pegs inside?

---

### Notes:

[BACK](#)

## Coloured Socks

---

*\*Credit to Martin Gardner in Entertaining Mathematical Puzzles*

### **Problem:**

Ten red socks and ten blue socks are all mixed up in a dresser drawer. The twenty socks are exactly alike except for their colour. The room is in pitch darkness and you want two matching socks. What is the smallest number of socks you must take out of the drawer in order to be certain that you have a pair that match?

---

### **Extension:**

What about two pair that match?

---

### **Notes:**

# No Change

---

*\*Credit to Martin Gardner in Entertaining Mathematical Puzzles*

## Problem:

“Give me change for a dollar, please,” said the customer.

“I’m sorry,” said Miss Jones, the cashier, after searching through the cash register, “but I can’t do it with the coins I have here.”

“Can you change half a dollar then?”

Miss Jones shook her head. In fact, she said, she couldn’t even make change for a quarter, dime, or nickel!

“Do you have any coins at all?” asked the customer.

“Oh yes,” said Miss Jones. “I have \$1.15 in coins.”

Exactly what coins were in the cash register?

---

## Extension:

---

## Notes:

[BACK](#)

# The Bicycles and the Fly

---

*\*Credit to Martin Gardner in Entertaining Mathematical Puzzles*

## Problem:

Two boys on bicycles, 20 km apart, began racing directly toward each other. The instant they started, a fly on the handle bar of one bicycle started flying straight toward the other cyclist. As soon as it reached the other handle bar it turned and started back. The fly flew back and forth in this way, from handle bar to handle bar, until the two bicycles met.

If each bicycle had a constant speed of 10 km an hour, and the fly flew at a constant speed of 15 km an hour, how far did the fly fly?

---

## Extension:

---

## Notes:

[BACK](#)

# The Floating Hat

---

*\*Credit to Martin Gardner in Entertaining Mathematical Puzzles*

## Problem:

A fisherman, wearing a large straw hat, was fishing from a rowboat in a river that flowed at a speed of three miles an hour. His boat drifted down the river at the same rate.

“I think I’ll row upstream a few miles,” he said to himself. “The fish don’t seem to be biting here.”

Just as he started to row, the wind blew off his hat and it fell into the water beside the boat. But the fisherman did not notice his hat was gone until he had rowed upstream five miles from his hat. Then he realized what must have happened, so he immediately started rowing back downstream again until he came to his floating hat.

In still water, the fisherman’s rowing speed is always five miles an hour. When he rowed upstream and back, he rowed at this same constant speed, but of course this would not be his speed relative to the shore of the river. For instance, when he rowed upstream at five miles an hour, the river would be carrying him downstream at three miles an hour, so he would be passing objects on the shore at only two miles an hour. And when he rowed downstream, his rowing speed and the speed of the river would combine to make his speed eight miles an hour with respect to the shore.

If the fisherman lost his hat at two o’clock in the afternoon, what time was it when he recovered it?

---

## Extension:

Make your own extension!

---

## Notes:

[BACK](#)

## Another Round Trip

---

*\*Credit to Martin Gardner in Entertaining Mathematical Puzzles*

### Problem:

When a trip is made by car, the car will of course travel different speeds at different times. If the total distance is divided by the total driving time, the result is called the *average speed* for that trip.

Mr. Smith planned to drive from Chicago to Detroit, then back again. He wanted to average 60 miles an hour for the entire round trip. After arriving in Detroit he found that his average speed for the trip was only 30 miles an hour.

What must Smith's average speed be on the return trip in order to raise his average speed for the round trip to 60 miles an hour?

---

### Extension:

Make your own extension!

---

### Notes:

[BACK](#)

## Cutting the Pie

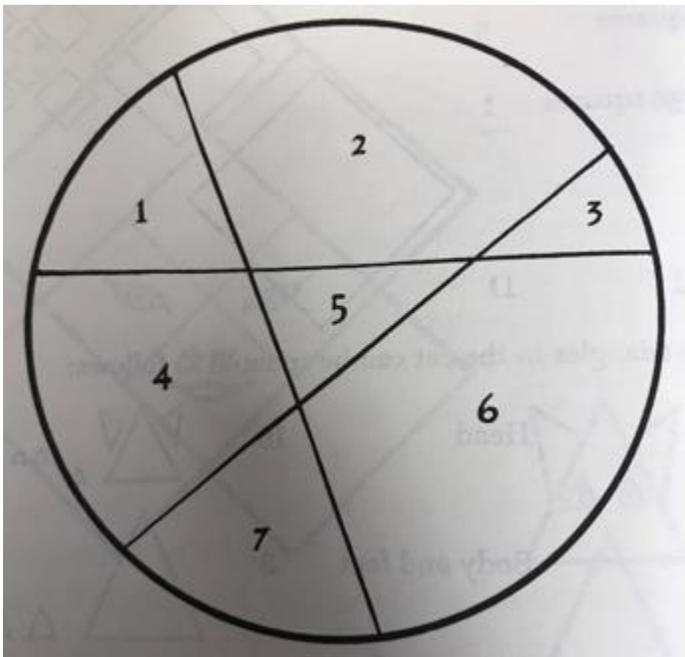
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*\*Credit to Martin Gardner in Entertaining Mathematical Puzzles*

### Problem:

With one straight cut you can slice a pie into two pieces. A second cut that crosses the first one will produce four pieces, and a third cut (see the illustration) can produce as many as seven pieces.

What is the largest number of pieces that you can get with six straight cuts?



---

### Extension:

Make your own extension!

---

### Notes:

[BACK](#)

# The Tallest Tower

---

*\*Credit to Peter Liljedahl*

## Problem:

With 10 pieces of paper and a box of paper clips, who can build the tallest tower?

---

## Extension:

Who can build the strongest tower?

---

## Notes:

Paper and paper clips.

[BACK](#)

# Bar Stools

---

*\*Credit to Peter Liljedahl*

## Problem:

My son noticed that he was the same height sitting at a bar stool as he was standing at the same bar stool. Explore and investigate.

---

## Extension:

Explore the mathematics behind body proportions, math in nature, and patterns.

---

## Notes:

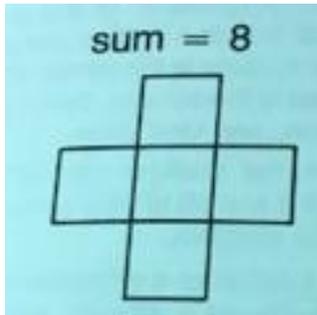
[BACK](#)

## Sum Sum Sum

*\*Credit to Dale Seymour and Favorite Problems*

### Problem:

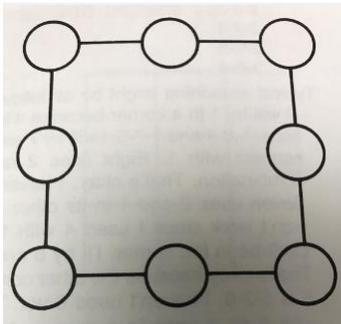
Place the numbers 1, 2, 3, 4, 5 in the squares so that the sum of the 3 numbers in a vertical or horizontal line equals the sum above that puzzle.



What if the sum was 9 or 10?

### Problem:

Place the numbers 1, 2, 3, 4, 5, 6, 7, 8 in the eight circles so that the sum of the numbers in any line equals 13.



Is this the only way? What other sums can you find? What sums can't you find?

### Extension:

### Notes:

[BACK](#)

## Prisoner in the Pool

---

*\*Credit to Vector 54(2) p. 95*

**Problem:** An escaped prisoner is in the middle of a square swimming pool. One of the guards chasing him is on land at one corner of the pool. The prisoner swims slower than the guard can run, but he can run faster than the guard can run. The guard does not swim. In which direction should the prisoner swim in order to increase his likelihood of getting away?

---

**Extension:**

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**Notes:**

[BACK](#)

# Birthdays

---

## Problem:

Something happens to my age every time I have a birthday. I get a year older, obviously. I noticed something else: I had my first birthday, then when I turned two, my age doubled. When I turned three, my age increased by 33.3...%.

What happened on my fourth birthday? My age increased by how much?  
What will happen at my 50th birthday?  
Is there a pattern year to year?

---

## Extension:

---

## Notes:

[BACK](#)

# Tanga Row

*\*Credit to Greg Tang @gregtangmath*

## Problem:

Use all the digits from 1 to 9 to make a multiplication problem.



TangaRow™

---


$$\begin{array}{r}
 \square \square \square \\
 \times \square \square \\
 \hline
 \square, \square \square \square
 \end{array}$$



## TangaRow™ Multiplication Puzzles

---

Use 7 consecutive digits – a TangaRow!

$$\square \square \times \square \square = \square \square \square$$

## Extension:

## Notes:

[BACK](#)

# A Staggering Pattern

---

*\*Credit to Problem of the Week by Linda Griffin and Glenda Demoss*

## Problem:

A	B	C	D	E
1	2	3	4	5
9	8	7	6	
10	11	12	13	14
18	17	16	15	

How can you continue this pattern?

---

## Extension:

What other numbers can you place?

---

## Notes:

# The King's Chessboard

---

*\*Credit to David Birch's book The King's Chessboard*

## Problem:

"You have served me well," said the King to the wise man. "What do you wish as a reward?"

The wise man was silent for a long time. And then the small wooden chessboard next to the King seemed to catch his interest. "Very well, sire," the wise man said at last. "I ask only this: Tomorrow, for the first square of your chessboard, give me one grain of rice; the next day, for the second square, two grains of rice; the next day after that, four grains of rice; then, the following day, eight grains for the next square of your chessboard. Thus for each square give me twice the number of grains of the square before it, and so on for every square of the chessboard."

Now the King wondered, as anyone would, just how many grains of rice this would be...

---

## Extension:

---

## Notes:

[BACK](#)

## What Does a Trillion Dollars Look Like?

---

### Problem:

What does a trillion dollars look like?

How much space would you need to store it?

---

### Extension:

---

### Notes:

Visit <http://www.pagetutor.com/trillion/index.html> for a visual.

[BACK](#)

## Classic Candle

---

### Problem:

With only a book of matches and a box of thumbtacks, how can you attach a lit candle to a wall (or cork board) in a way so the candle wax won't drip onto the table below?

---

### Extension:

---

### Notes:

[BACK](#)

# Our Future

---

*\*Credit to Maisha Winn*

## Problem:

- There are 78 000 000 people under the age of 18 in our country.
- Almost 25% of the nation's population.
- 50 000 000 are in schools (1.3 million are homeless)

How can we teach mathematics so that people stop hating and killing one another?

Explore a social justice issue with mathematics. How can we use math to make the world a better place?

---

## Extension:

Polution, Famine, War

---

## Notes:

This is American data.

[BACK](#)